Vooludünaamika laiendatud geomeetrias

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28. oktoober 2014

Fluid dynamics
on generalized geometric backgrounds

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Outline

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Fluids are everywhere

- Perfect fluid:
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
    - Dark energy: $p < -\frac{1}{3}\rho$.
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- Maxwell-Boltzmann gas:
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  - Used in structure formation, atmosphere dynamics...
- Charged, multi-component gas:
  - Plasma, interacting gas including recombination / ionization.
  - Used in stellar dynamics, pre-CMB era models...

From spacetime to observer space

- Fluid dynamics naturally lift to tangent bundle:
    - Fluids conveniently modeled by particle dynamics (SPH. . .).
    - Physical fluids constituted by particles.
    - Particle trajectories lift to tangent bundle: $\gamma \rightsquigarrow (\gamma, \dot{\gamma})$.

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  - Measured tensor components depend on observer velocity.
  - Physical observer velocities are future unit timelike vectors.

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- Space of observers corresponds to particle tangent vectors.

$\Rightarrow$ Consider fluid dynamics on observer space!

Finsler spacetimes

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  - Other matter field theories
- Possible explanations of yet unexplained phenomena:
  - Fly-by anomaly
  - Galaxy rotation curves
  - Accelerating expansion of the universe

Outline

The clock postulate

- Proper time along a curve in Lorentzian spacetime:

$$\tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)}dt\,.$$

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- Finsler geometry: use a more general length functional:

$$\tau = \int_{t_1}^{t_2} F(x(t), \dot{x}(t))dt\,.$$

- Finsler function $F : TM \to \mathbb{R}^+$.
- Parametrization invariance requires homogeneity:

$$F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0\,.$$

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

⇒ Finsler metric with Lorentz signature:

$$g^F_{ab}(x,y) = \frac{1}{2}\frac{\partial}{\partial y^a}\frac{\partial}{\partial y^b}F^2(x,y)\,.$$

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- Unit vectors $y \in T_xM$ defined by

$$F^2(x,y) = g^F_{ab}(x,y)y^ay^b = 1\,.$$

⇒ Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.

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- Ω_x contains a closed connected component $S_x \subseteq \Omega_x$.

$\rightsquigarrow$ Causality: S_x corresponds to physical observers.

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]$$

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⇒ Split of the tangent and cotangent bundles:

- Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a \bar{\partial}_b \,, \quad \bar{\partial}_a$$

- Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

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- Geodesic spray:

$$\mathbf{S} = y^a\delta_a$$

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
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- Definition of observer space:

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- Geodesic hypersurface measure $\omega = \iota_{\mathbf{r}}\Sigma$.
- Note that $\mathcal{L}_{\mathbf{r}}\Sigma = 0$ and $d\omega = 0$.

From metric to Finsler geometry

Tangent bundle geometry:

- Finsler function:

$$F(x, y) = \sqrt{|g_{ab}(x) y^a y^b|}$$

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$$g^F_{ab}(x, y) = \begin{cases} -g_{ab}(x) & y \text{ timelike} \\ g_{ab}(x) & y \text{ spacelike} \end{cases}$$

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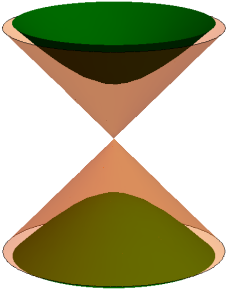
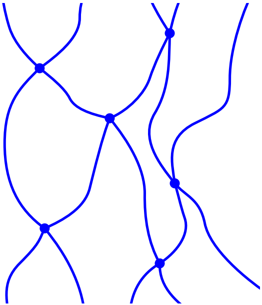


- Observer space:
  - Space Ω_x of unit timelike vectors at $x \in M$.
  - Space S_x of future unit timelike vectors at $x \in M$.
  - Observer space O: union of shells S_x.

Outline

Definition of fluids

- Single-component fluid:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.



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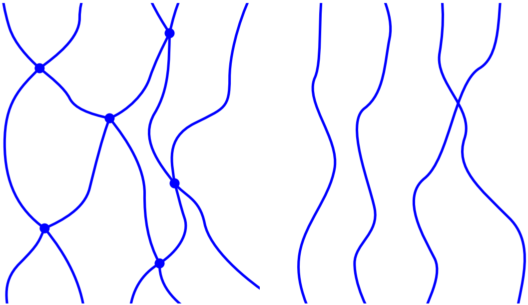
  - $\Rightarrow$ Particles follow geodesics.



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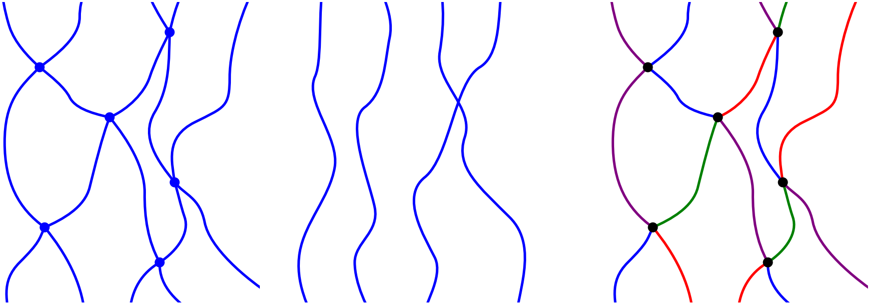
- Multi-component fluid: multiple types of particles.



Geodesics on observer space

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

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- Canonical lift of curve to tangent bundle TM:

$$x\,, \quad y = \dot{x}\,.$$

- Lift of geodesic equation:

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- Tangent vectors are future unit timelike: $(x, y) \in O$.

⇒ Particle trajectories are piecewise integral curves of **r** on O.

One-particle distribution function

- Recall: $\omega = \iota_{\mathbf{r}}\Sigma \in \Omega^6(O)$ unique 6-form such that:
  - ω non-degenerate on every hypersurface not tangent to $\mathbf{r}$.
  - $d\omega = 0$.

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  - ω non-degenerate on every hypersurface not tangent to $\mathbf{r}$.
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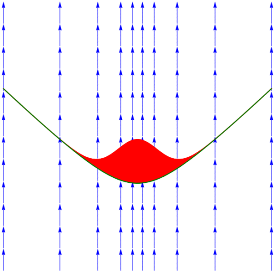
- Define one-particle distribution function $\phi : O \to \mathbb{R}^+$ such that:

For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi\omega$$

# of particle trajectories through σ.



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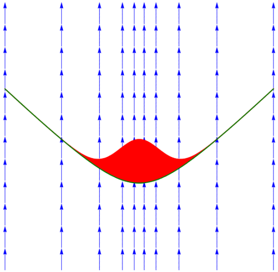

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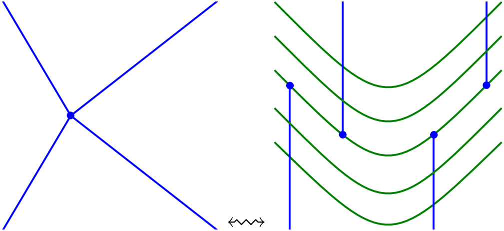
  - Counting of particle trajectories respects hypersurface orientation.
- For multi-component fluids: ϕ_i for each component i.

Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



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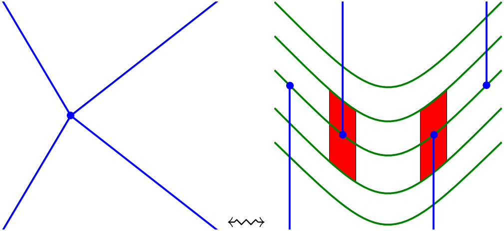

- For any open set $V \in O$,

$$\int_{\partial V} \phi\omega = \int_V d(\phi\omega) = \int_V \mathcal{L}_{\mathbf{r}}\phi\Sigma$$

# of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}}\phi$.

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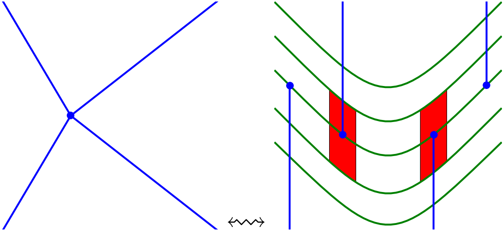

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$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}}\phi$.

- Collisionless fluid: trajectories have no endpoints, $\mathcal{L}_{\mathbf{r}}\phi = 0$.

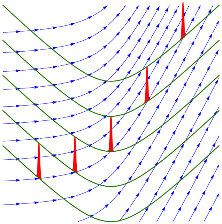
$\Rightarrow$ Simple, first order equation of motion for collisionless fluid.

$\Rightarrow$ ϕ is constant along integral curves of $\mathbf{r}$.

Examples of fluids

Geodesic dust fluid:

$$\phi(x,y) \sim \delta(y - u(x)) \,.$$



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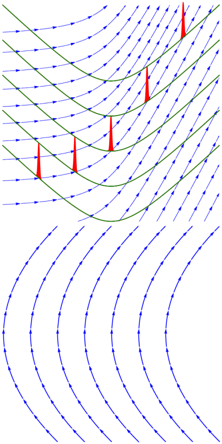

"Jenkka"

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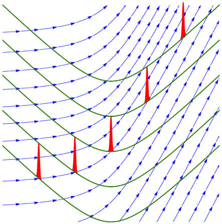

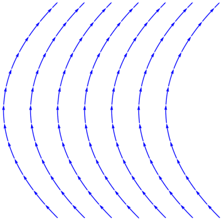

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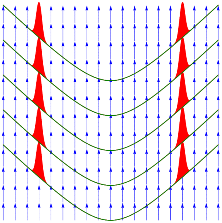

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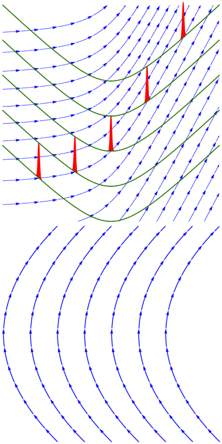

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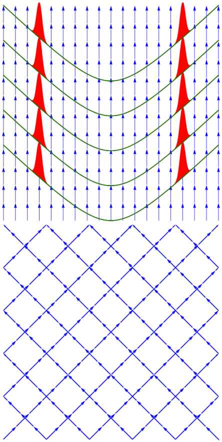

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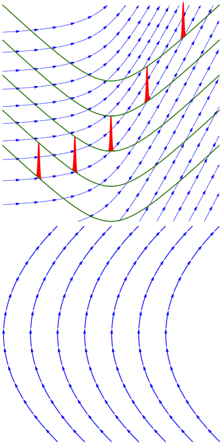

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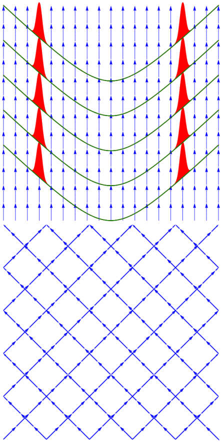
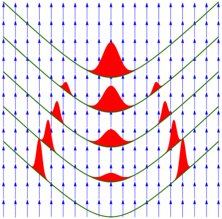


“Polkka”

Interacting fluid:

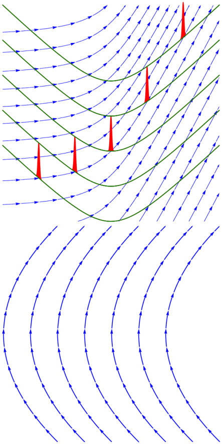
$$\mathcal{L}_{\mathbf{r}}\phi \neq 0\,.$$



Examples of fluids

Geodesic dust fluid:

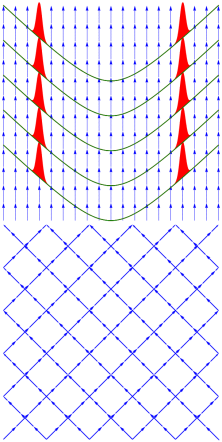
$$\phi(x, y) \sim \delta(y - u(x)) .$$



"Jenkka"

Collisionless fluid:

$$\mathcal{L}_{\mathbf{r}}\phi = 0 .$$



"Polkka"

Interacting fluid:

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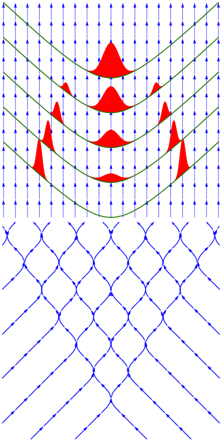

"Humppa"

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$\Rightarrow$ Connection to well-known spacetime observables.

$\Rightarrow$ Connection to measurements.

Symmetric solutions

- Infinitesimal diffeomorphism described by vector field ξ on M.
- Canonical lift of ξ to vector field on TM:

$$\hat{\xi} = \xi^a \partial_a + y^a \partial_a \xi^b \bar{\partial}_b$$

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- Symmetry provides simplification of 7-dimensional O:
  - Spherical symmetry: 4 dimensions remain.
  - Static spherical symmetry: 3 dimensions remain.
  - Cosmological symmetry: 2 dimensions remain.

Outline

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  - Energy-momentum tensor
- Symmetries defined “as usual” by Killing vector fields.

Outlook

- Coupling of fluids to non-metric gravity theories.
- Cosmological solutions with non-metric geometry.
- Extension of parameterized post-Newtonian formalism.
- ...

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