Finsler spacetimes with spherical and cosmological symmetry

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Outline

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Motivation

- Modify spacetime geometry to address open problems:
  - Origin of dark matter and dark energy.
  - Homogeneity of the cosmic microwave background and inflation.
  - Fly-by anomaly in the solar system.

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  - Divide tangent spaces into space-, time-, lightlike vectors.
  - Provide notions of future and past.
  - Distinguish curves corresponding to physical trajectories.
  - Define proper time along physical trajectories.
  - Determine trajectories of freely falling test masses.
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  - Finsler geodesics determine notion of free fall.
  - Gravity theory on Finsler spacetimes exists.

From metric to Finsler geometry

The clock postulate

Proper time along a curve in Lorentzian spacetime:

$$\tau[\gamma] = \int_{t_1}^{t_2} \sqrt{-g_{\mu\nu}(\gamma(t))\dot{\gamma}^\mu(t)\dot{\gamma}^\nu(t)}dt\,.$$

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Generalized clock postulate: Finsler length measure

- Finsler geometry: use a more general length functional:

$$\tau[\gamma] = \int_{t_1}^{t_2} F(\gamma(t), \dot{\gamma}(t))dt\,.$$

- Finsler function $F : TM \to \mathbb{R}^+$.
- Parametrization invariance requires homogeneity:

$$F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0\,.$$

Finsler geodesics

Cartan non-linear connection

- Extremal curve of length functional satisfies geodesic equation:

$$\ddot{\gamma}^\mu(t) + N^\mu{}_\nu(\gamma(t), \dot{\gamma}(t)) = 0\,.$$

- $N^\mu{}_\nu$: coefficients of Cartan non-linear connection.
- Horizontal-vertical split of $TTM = HTM \oplus VTM$:

$$\delta_\mu = \frac{\partial}{\partial x^\mu} - N^\nu{}_\mu \frac{\partial}{\partial y^\nu}\,, \quad \bar{\partial}_\mu = \frac{\partial}{\partial y^\mu}\,.$$

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Geodesic spray

- Canonical lift $\Gamma^a = (\gamma^\mu, \dot{\gamma}^\mu)$ of geodesic to TM satisfies

$$\dot{\Gamma}^a(t) - \mathbf{S}^a(\Gamma(t)) = 0$$

- $\mathbf{S}(x, y) = y^\mu \delta_\mu \in \mathrm{Vect}(TM)$: geodesic spray.

Finsler gravity action and field equations

Finsler gravity action

$$S_G = \int_\Sigma \mathcal{R} \, \mathrm{Vol}(G|_\Sigma) \,.$$

- Σ: Unit tangent bundle $TM|_{F=1}$.
- G: Sasaki metric on TM.
- $\mathcal{R}$: Scalar curvature of Cartan non-linear connection.

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Gravitational field equations [Pfeifer, Wohlfarth '11]

$$-\frac{1}{F^2}\left\{6\mathcal{R} + G^{ab}\left[\nabla^v_a\nabla^v_b\mathcal{R} + 2F^2 J^c{}_a\nabla^h_b \$_c + 2\nabla^v_a\left(\mathbf{S}^c\nabla^h_c \$_b\right)\right]\right\} = \mathcal{T}\,.$$

- $a, b, c, \ldots$: Coordinate indices $0, \ldots, 7$ on TM.
- $J^a{}_b$: Tangent structure.
- $\mathbf{S}^a$: Geodesic spray.
- $\$_a$: Landsberg covector.
- ∇^h, ∇^v: Horizontal and vertical Berwald derivative.
- $\mathcal{T}$: Energy-momentum scalar.

Observer space

Finsler metric

- Metric structure on Finsler spacetimes:

$$g^F_{\mu\nu} = \frac{1}{2}\bar{\partial}_\mu\bar{\partial}_\nu F^2 \,.$$

- Finsler metric $g^F_{\mu\nu}$ has Lorentz signature.

$\Rightarrow$ Definition of timelike, lightlike, spacelike tangent vectors.

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Observer space

- Closed shell $S_x \subset T_xM$ of future unit timelike vectors for all $x \in M$.
- Space of physical observer velocities:

$$O = \bigcup_{x \in M} S_x \,.$$

Relation to Cartan geometry

Observer frame bundle

- Set P of frames f of TM such that:
  - Time component $f_0 \in O$.
  - Frame is orthonormal with respect to Finsler metric:

$$f^{\mu}_{\alpha} f^{\nu}_{\beta} g^{F}_{\mu\nu} = -\eta_{\alpha\beta} \, .$$

- Principal SO(3) bundle $\pi : P \to O, f \mapsto f_0$.

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Cartan connection [MH '13]

- Cartan connection $A = \omega + e \in \Omega^1(P, \mathfrak{g})$ with $G = \mathrm{ISO}(3,1)$:

$$e^\alpha = f^{-1}{}^\alpha{}_\mu dx^\mu\,,$$

$$\omega^\beta{}_\alpha = \frac{1}{2}\left(\delta^\gamma_\alpha\delta^\beta_\delta - \eta^{\beta\gamma}\eta_{\alpha\delta}\right) f^{-1}{}^\delta{}_\mu df^\mu_\gamma + \frac{1}{2}\eta^{\beta\gamma} f^\nu_\alpha f^\sigma_\gamma(\delta_\nu g^F_{\mu\sigma} - \delta_\sigma g^F_{\mu\nu})dx^\mu\,.$$

- Finsler gravity action can be written completely in terms of A.

Outline

Cosmological symmetry

Cosmological coordinates on TM [MH '15]

- Spherical coordinates t, r, ϑ, φ on M.
- Coordinates y, u, v, w on each T_xM:

$$y\partial_t + w\left[\cos u\sqrt{1 - kr^2}\partial_r + \frac{\sin u}{r}\left(\cos v\partial_\vartheta + \frac{\sin v}{\sin\vartheta}\partial_\varphi\right)\right] \in T_xM\,.$$

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Cosmologically symmetric Finsler spacetime

- Symmetry under rotations and translations (six vector fields).
- Most general Finsler function: $F(t, y, w)$.
- Homogeneity condition: $F(t, \lambda y, \lambda w) = \lambda F(t, y, w)$.
- Express Finsler function as $F(t, y, w) = y\tilde{F}(t, w/y)$.

Observer space coordinates

Observer trajectories

- Tangent vectors are future unit timelike vectors: $F = 1$.

$\Rightarrow$ Physical tangent vectors lie in observer space O.

- Introduce suitable coordinates on observer space.

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Observer space coordinates [MH '15]

- Introduce coordinates:

$$T = t, R = r, \Theta = \vartheta, \Phi = \varphi, Y = y\tilde{F}\left(t, \frac{w}{y}\right), U = u, V = v, W = w/y\,.$$

$\Rightarrow$ Observer space is submanifold with $Y = 1$.

$\Rightarrow$ Coordinates become singular on light cone, since $Y = 0$.

Geodesics on cosmological background

Radial geodesic

- Consider radial motion: $\vartheta = \pi/2, \varphi = 0, u = 0, v = 0$.
- Geodesic equation:

$$\dot{t} = y\,, \quad \dot{r} = w\sqrt{1 - kr^2}\,,$$

$$\dot{y} = -y^2 \frac{\tilde{F}_{ww}\tilde{F}_t - \tilde{F}_w\tilde{F}_{tw}}{\tilde{F}\tilde{F}_{ww}}\,, \quad \dot{w} = -y\frac{w\tilde{F}_t\tilde{F}_{ww} + y\tilde{F}\tilde{F}_{tw} - w\tilde{F}_w\tilde{F}_{tw}}{\tilde{F}\tilde{F}_{ww}}\,.$$

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Timelike radial geodesic

- Geodesic equation in observer coordinates:

$$\dot{T} = \frac{Y}{\tilde{F}}\,, \quad \dot{R} = \frac{WY}{\tilde{F}}\sqrt{1 - kR^2}\,, \quad \dot{Y} = 0\,, \quad \dot{W} = -\frac{Y\tilde{F}_{tw}}{\tilde{F}\tilde{F}_{ww}}\,.$$

- Use arc length parametrization and fix $Y = 1$:

$$\dot{T} = \frac{1}{\tilde{F}}\,, \quad \dot{R} = \frac{W}{\tilde{F}}\sqrt{1 - kR^2}\,, \quad \dot{W} = -\frac{\tilde{F}_{tw}}{\tilde{F}\tilde{F}_{ww}}\,.$$

Fluid dynamics with cosmological symmetry

Kinetic theory of fluids [Ehlers '71], [Sarbach, Zannias '13]

- Consider fluid as constituted by point particles.
- Particles follow piecewise geodesics between collisions.
- Continuum limit described by density $\phi : O \to \mathbb{R}^+$.
- Collisionless fluid satisfies Liouville equation $\mathcal{L}_{\mathbf{S}}\phi = 0$.

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Cosmologically symmetric Finsler fluids [MH '15]

- Most general cosmologically symmetric fluid: $\phi = \phi(T, W)$.
- Liouville equation: $\phi_t \tilde{F}_{ww} = \phi_w \tilde{F}_{tw}$.
- Example: collisionless dust fluid $\phi(x, y) \sim \rho(x)\delta_{S_x}(y, u(x))$:

$$u(t) = \frac{1}{\tilde{F}(t, 0)}\partial_t\,, \quad \partial_t\left(\rho(t)\sqrt{g^F(t, 0)}\right) = 0\,.$$

Gravitational dynamics

Finsler gravity [Pfeifer, Wohlfarth '11]

- Action:

$$S_G = \int_\Sigma \mathcal{R}\,\mathrm{Vol}(G|_\Sigma)\,.$$

- Field equations:

$$-\frac{1}{F^2}\left\{6\mathcal{R} + G^{ab}\left[\nabla^v_a\nabla^v_b\mathcal{R} + 2F^2 J^c{}_a\nabla^h_b \$_c + 2\nabla^v_a\left(\mathbf{S}^c\nabla^h_c \$_b\right)\right]\right\} = \mathcal{T}\,.$$

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Cosmological dynamics

- Structure of cosmological equations: $\mathcal{G}[\tilde{F}](T, W) = \mathcal{T}[\tilde{F}, \phi](T, W)$.
- Difficulties:
  - Geometry scalar $\mathcal{G}$ is complicated even for cosmology.
  - No "standard construction" for $\mathcal{T}$ of non-metric kinetic fluid.

Example: FLRW spacetime

Geometry

- Tensor field: metric $g_{\mu\nu}$.
- Cosmology: FLRW metric $g = -dt \otimes dt + a^2(t)\gamma_{ij}[\kappa]dx^i \otimes dx^j$.
- Finsler function:

$$\tilde{F}(x, y) = \sqrt{|g_{\mu\nu}y^\mu y^\nu|} = \sqrt{\left|1 - a^2(T)W^2\right|}$$

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Gravitational dynamics

- Geometry scalar:

$$\mathcal{G} = \frac{6}{a^2(1 - W^2 a^2)}\left(a\ddot{a} - 2\dot{a}^2 - 2\kappa + W^2 a^3 \ddot{a}\right) .$$

$\Rightarrow$ Reproduce structure of Friedmann equations.

Example: cosmologies with one-forms

Ingredients

- Tensor fields: metric $g_{\mu\nu}$, one-form A_μ.
- Cosmology:
  - FLRW metric $g = -dt \otimes dt + a^2(t)\gamma_{ij}[\kappa]dx^i \otimes dx^j$.
  - Hypersurface normal $A = b(t)dt$.

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Randers length measure [Randers '41]

$$\tilde{F}(x,y) = \sqrt{|g_{\mu\nu}y^\mu y^\nu|} + A_\mu y^\mu = \sqrt{|1 - a^2(T)W^2|} + b(T)$$

Bogoslovsky length measure [Bogoslovsky '77]

$$\tilde{F}(x,y) = (A_\mu y^\mu)^q \left(\sqrt{|g_{\mu\nu}y^\mu y^\nu|}\right)^{1-q} = b^q(T)\left(\sqrt{|1 - a^2(T)W^2|}\right)^{1-q}$$

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Spherical symmetry

Spherical coordinates on TM

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- Coordinates y, u, v, w on each T_xM:

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Spherically symmetric Finsler spacetime

- Symmetry under rotations around origin (three vector fields).
- Most general Finsler function: $F(t, r, y, u, w)$.
- Homogeneity condition: $F(t, r, \lambda y, \lambda u, \lambda w) = \lambda F(t, r, y, u, w)$.
- Express Finsler function as $F(t, r, y, u, w) = y\tilde{F}(t, r, u/y, w/y)$.
- Static case: $F(r, y, u, w) = y\tilde{F}(r, u/y, w/y)$.

Example: static circular orbits

Circular geodesic motion

- Circular motion: $\vartheta = \pi/2, u = 0, v = \pi/2$.
- Orbit condition: $w\tilde{F}_w + ry\tilde{F}_r = 0$.
- Geodesic equation: $\dot{t} = y, \dot{\varphi} = w/r$.
- Orbital period: $2\pi ry/w$.

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Example: Schwarzschild spacetime

- Schwarzschild length function:

$$\tilde{F}(R, U, W) = \sqrt{1 - \frac{2M}{R} - \frac{U^2}{1 - \frac{2M}{R}} - W^2}\,.$$

- Orbit condition: $My^2 = rw^2$.
- Orbital period: $2\pi\sqrt{r^3/M}$.

Outline

1. Introduction
2. Cosmological symmetry
3. Spherical symmetry
4. **Conclusion**

Summary

- Finsler spacetimes:
  - Based on Finsler length function.
  - Make use of tensors on the tangent bundle.
  - Generalize standard notions of causality, observers and gravity.
- Cosmologically symmetric Finsler spacetimes:
  - Geometry defined by function $\tilde{F}(T, W)$.
  - Simple form of geodesic equation.
  - Simple equation of motion for fluid dynamics.
  - Gravitational field equations are rather complicated.
  - Simple examples can be derived from tensorial geometries.
- Spherically symmetric Finsler spacetimes:
  - Geometry defined by function $\tilde{F}(T, R, U, W)$.
  - Static geometry reduces to $\tilde{F}(R, U, W)$.
  - Simple condition for circular orbits.

Outlook

- Finsler fluid dynamics:
  - Derive dynamics for well-known types of fluids.
  - Construct energy-momentum scalar for general kinetic fluid.
- Cosmologically symmetric Finsler spacetimes:
  - Find cosmologically symmetric solutions.
  - Calculate luminosity-redshift relation.
  - Calculate cosmological parameters.
- Spherically symmetric Finsler spacetimes:
  - Find spherically symmetric vacuum solutions.
  - Calculate analogue of post-Newtonian limit.
  - Calculate geodesic motion and address fly-by anomaly.
  - Investigate whether Birkhoff theorem holds.

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