Beyond fluids

Generalized matter models and their gravitational interaction

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Center of Excellence “The Dark Side of the Universe”

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Outline

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Problems in gravity and cosmology

- So far unexplained cosmological observations:
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  - Similarly: replacing flat spacetime by curved spacetime led to GR.
  - Replace perfect fluid model by velocity-dependent distribution of particles.
- Questions arising from new matter model:
  - ✓ How does a kinetic gas react to a gravitational field?
  - ? How does a kinetic gas create a gravitational field?

Examples of fluids

- Perfect fluid:
  - Most general energy-momentum tensor compatible with cosmological symmetry.
  - No shear stress, no friction.
  - Characterized by density ρ and pressure p.
    - Dust, dark matter: $p = 0$.
    - Radiation: $p = \frac{1}{3}\rho$.
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- Hyperfluid:
  - Additional coupling to affine connection generates hypermomentum.
  - Intrinsic property of matter, e.g., spin.

Why study matter beyond fluids?

- Dynamical friction:
  - Massive object passes distribution of light objects.
  - ⇒ Gravity of massive object changes positions of lighter objects.
  - ⇒ Perturbation of light objects asserts gravity on massive object.
  - Example: globular cluster passing through galaxy.

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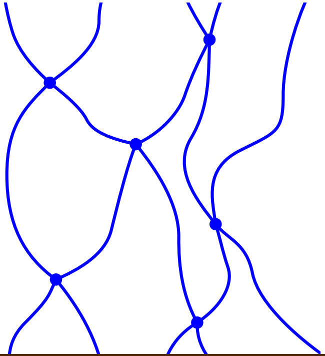
- Dynamics of intergalactic medium:
  - Cosmic gas highways: gas in and near filaments
  - Crossing sheets in collapse and structure formation.

Outline

Definition of kinetic gas

- Single-component gas:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.



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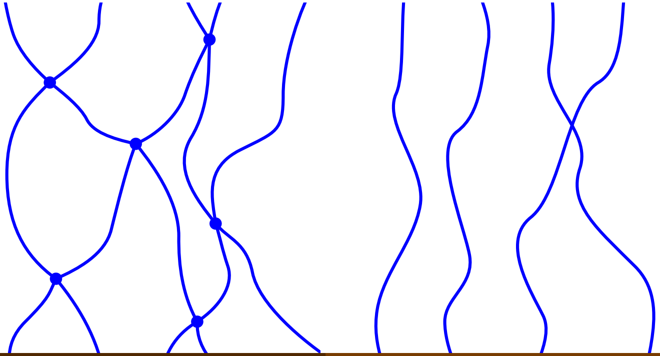

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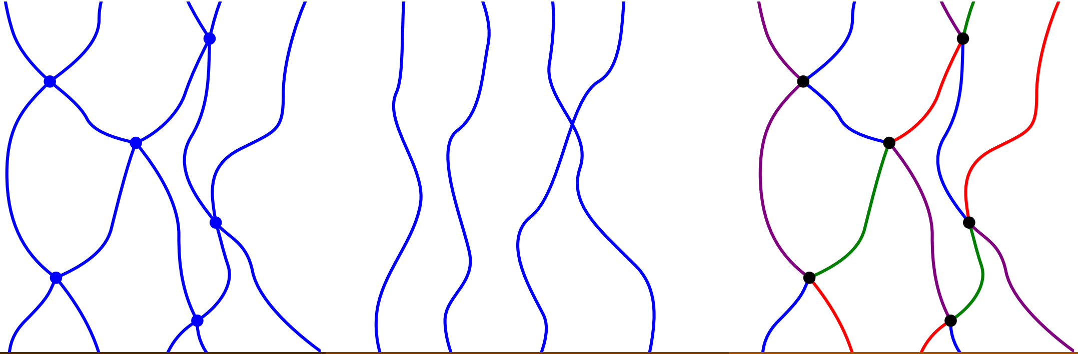
- Multi-component gas: multiple types of particles.



One-particle distribution function

- Kinetic gas described by density in velocity space:
  - Consider space O of physical (unit, timelike, future pointing) four-velocities.
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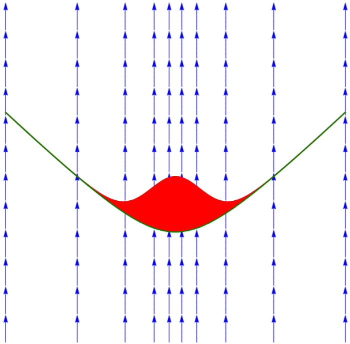
- Define one-particle distribution function $\phi : O \to \mathbb{R}^+$ such that:



For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_{\sigma} \phi \Omega$$

# of particle trajectories through σ.

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  - Counting of particle trajectories respects hypersurface orientation.

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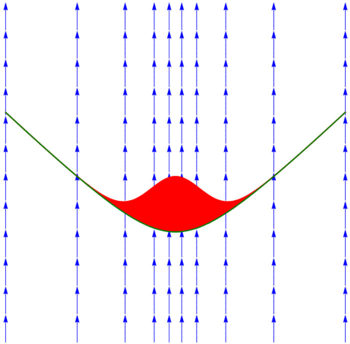

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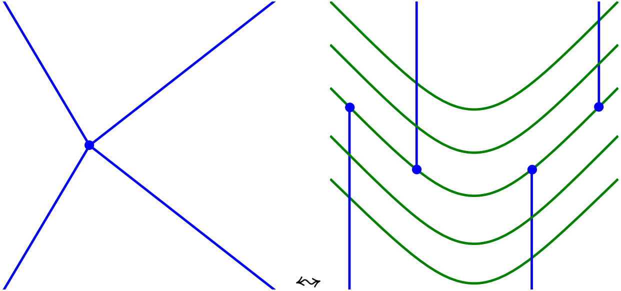
- For multi-component fluids: ϕ_i for each component i.

Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



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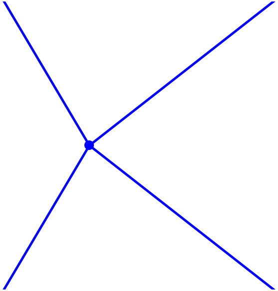
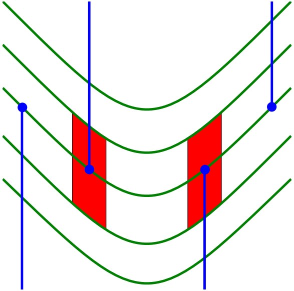


$\leftrightsquigarrow$



- For any open set $V \in O$,

$$\int_{\partial V} \phi \Omega = \int_V d(\phi \Omega) = \int_V \mathcal{L}_{\mathbf{r}} \phi \Sigma$$

# of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}} \phi$.

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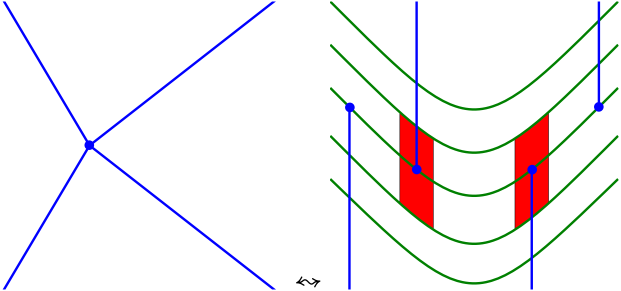

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- Collisionless fluid: trajectories have no endpoints, $\mathcal{L}_{\mathbf{r}}\phi = 0$.

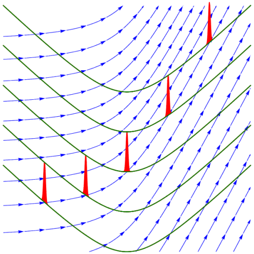
$\Rightarrow$ Simple, first order equation of motion for collisionless fluid.

$\Rightarrow$ ϕ is constant along integral curves of **r**.

Some (very) pictorial examples

Geodesic dust fluid:

$$\phi(x, y) \sim \delta(y - u(x)) .$$



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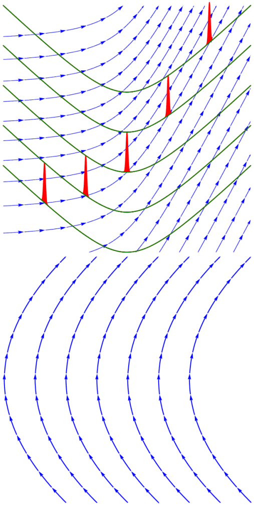

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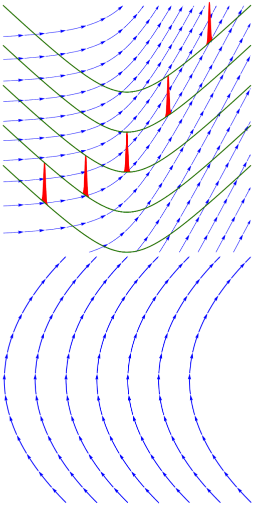

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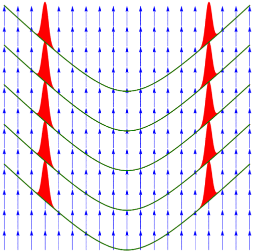

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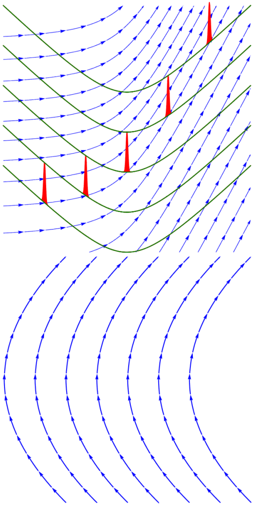

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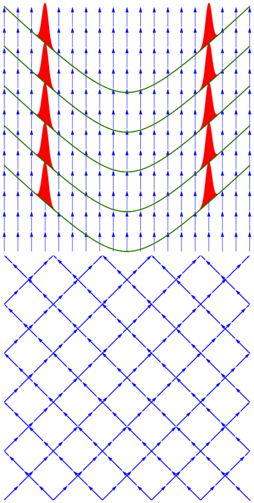

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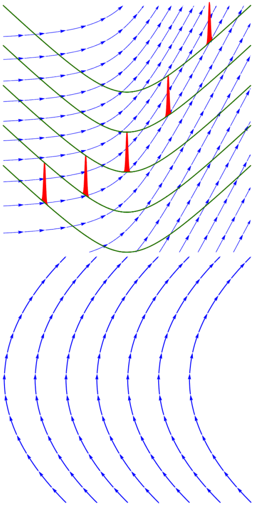

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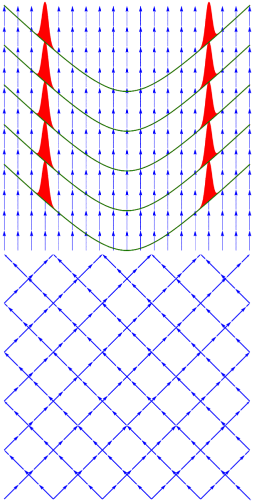

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Interacting fluid:

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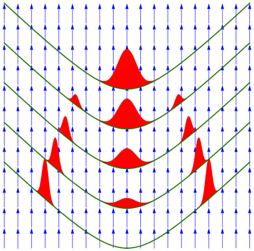

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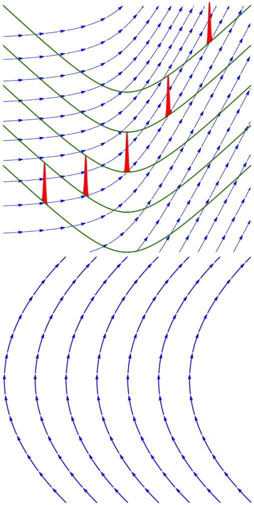

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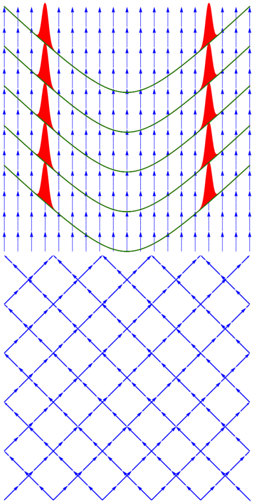

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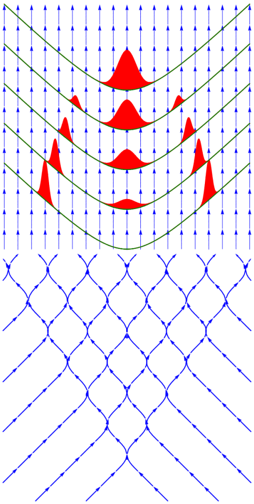

"Humppa"

Example: collisionless dust fluid

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⇒ Generalized (pressureless) Euler equations to Finsler geometry [MH '15].

- Metric limit $F^2(x, y) = |g_{ab}(x) y^a y^b|$ yields Euler equations:

$$u^b \nabla_b u^a = 0 , \quad \nabla_a(\rho u^a) = 0 .$$

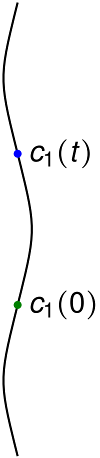
Outline

Action for a single point particle:

$$S = m \int_0^t (F \circ c_1)(\tau)\, d\tau\,.$$

Assume arc length parameter τ:

$$S = mt\,.$$



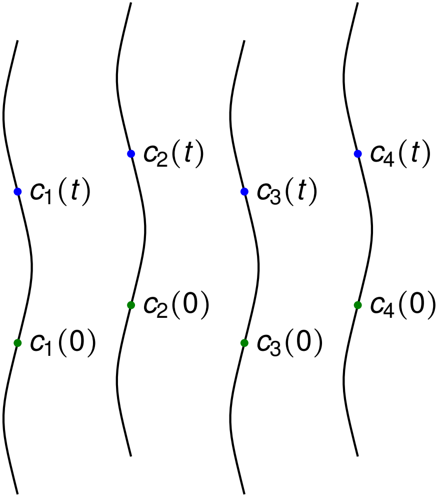
Action of a kinetic gas

Action for P point particles:

$$S_{\text{gas}} = m \sum_{i=1}^{P} \int_0^t (F \circ c_i)(\tau)\, d\tau\,.$$

Assume arc length parameter τ:

$$S_{\text{gas}} = Pmt\,.$$



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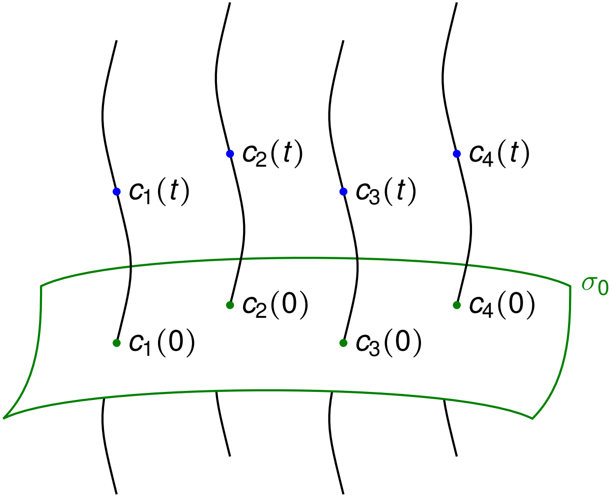
- Hypersurface of starting points:

$$c_i(0) \in \sigma_0 \,.$$



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- Hypersurface of end points:

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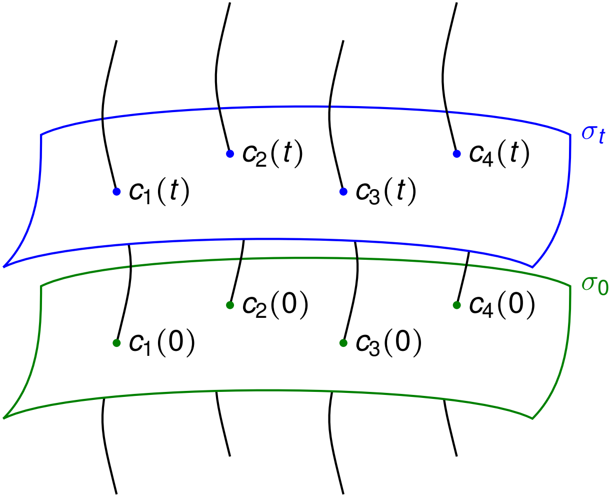

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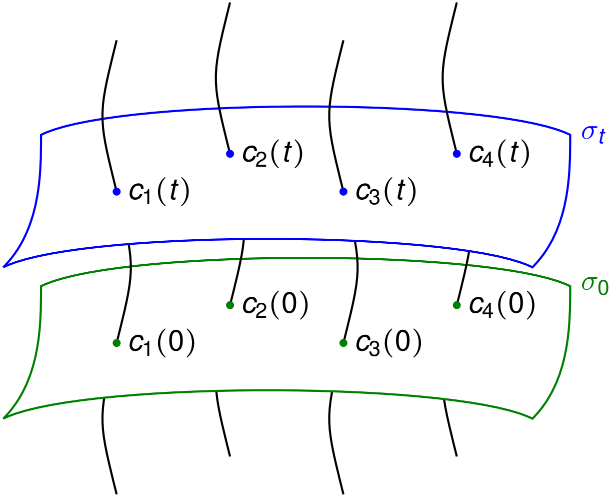
- Number of particle trajectories:

$$P = N[\sigma_\tau] = \int_{\sigma_\tau} \phi\Omega \,.$$



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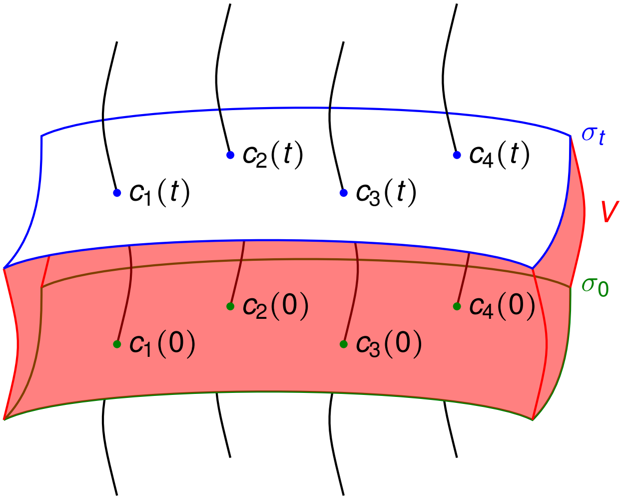
$$V = \bigcup_{\tau=0}^{t} \sigma_\tau \,.$$



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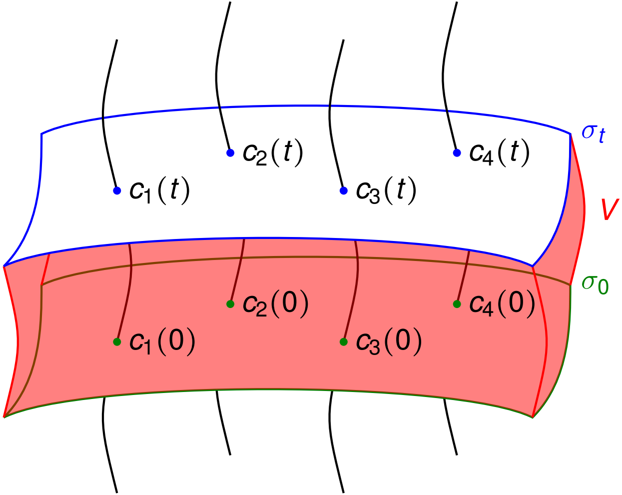
- Recall particle action integral:

$$S_{\text{gas}} = Pmt = m\int_0^t \left(\int_{\sigma_\tau} \phi\Omega\right) d\tau = m\int_V \phi\Omega \wedge \omega = m\int_V \phi\Sigma\,.$$

Defined through 1-PDF ϕ [MH, Pfeifer, Voicu '19].



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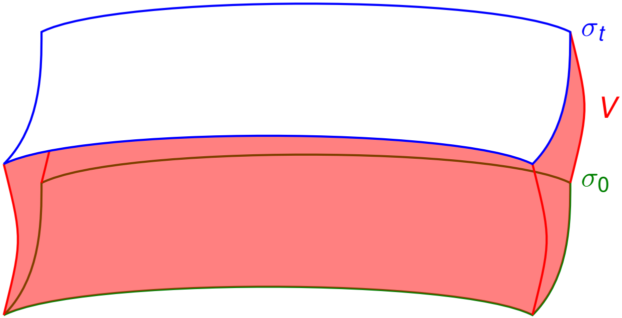
[MH, Pfeifer, Voicu '19].

$\Rightarrow$ Forget particle trajectories!



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⇒ Reduces to Einstein-Hilbert action for metric geometry.

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⇒ Gravitational field equations with kinetic gas matter [MH, Pfeifer, Voicu '19]:

$$\frac{1}{2} g^{F\,ab} \bar{\partial}_a \bar{\partial}_b (F^2 R_0) - 3R_0 - g^{F\,ab} (\nabla_{\delta_a} P_b - P_a P_b + \bar{\partial}_a (\nabla P_b)) = -\kappa^2 \phi$$

Physical implications

- There are no metric non-vacuum solutions to the field equations.
  - Field equations in case of a metric geometry $F^2 = g_{ab}(x)y^a y^b$:

$$3r_{ab}(x)y^a y^b - r(x)g_{ab}(x)y^a y^b = -\kappa^2\phi g_{ab}(x)y^a y^b\,.$$

  - Second derivative with respect to velocities y^a and y^b:

$$3r_{ab}(x) - r(x)g_{ab}(x) = -\kappa^2\phi g_{ab}(x)\,.$$

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  ↯ Unphysical velocity distribution: uniform over all (arbitrarily high) velocities!

⇒ Gravitational field of a kinetic gas always depends on the velocity of the observer.
  - For observers whose velocity exceeds that of any gas particles:

$$\frac{1}{2} g^{F\,ab} \bar{\partial}_a \bar{\partial}_b (F^2 R_0) - 3R_0 - g^{F\,ab}(\nabla_{\delta_a} P_b - P_a P_b + \bar{\partial}_a(\nabla P_b)) \to 0$$

  - Solution of the differential equation still depends on ϕ via boundary conditions.

  ⇒ Observers at velocities beyond gas velocities are still affected, but differently.

Outline

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- Homogeneity of Finsler function $F(t, y^t, w) = y^t \tilde{F}(t, w/y^t)$.
- Introduce new coordinates: $\tilde{y} = y^t \tilde{F}(t, w/y^t)$, $\tilde{w} = w/y^t$.

$\Rightarrow$ Coordinates on observer space O with $\tilde{y} \equiv 1$.

$\Rightarrow$ Geometry function $\tilde{F}(t, \tilde{w})$ on O.

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- Next task: solve cosmological field equations with kinetic gas.

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Conclusion

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  - Kinetic gas dynamics:
    - ⋆ Model many-particle systems defined by individual point mass trajectories.
    - ⋆ Consider space O of physical four-velocities (future unit timelike vectors).
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- Outlook:
  - Cosmological solutions with non-metric geometry: Dark energy? Inflation?
  - Weak field limit: Newtonian, post-Newtonian...
  - Dynamical friction?
  - Stellar streams?
  - Dynamics of heterogeneous systems: stars + gas in galaxies?

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