Modified dispersion relations and relativistic gas dynamics

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Manuel Hohmann

Laboratory of Theoretical Physics, Institute of Physics, University of Tartu
Center of Excellence "Fundamental Universe"

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$\Rightarrow$ Here: effective quantum gravity phenomenology with gas dynamics near black holes.

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$\rightsquigarrow$ Only need to study (all) possible quantum corrections!

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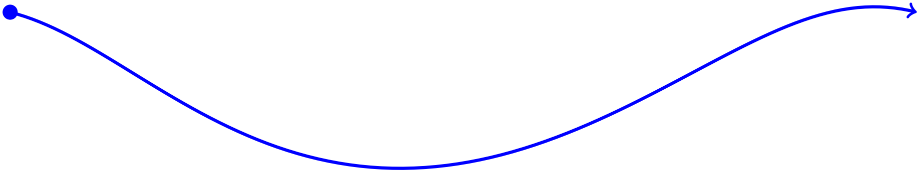

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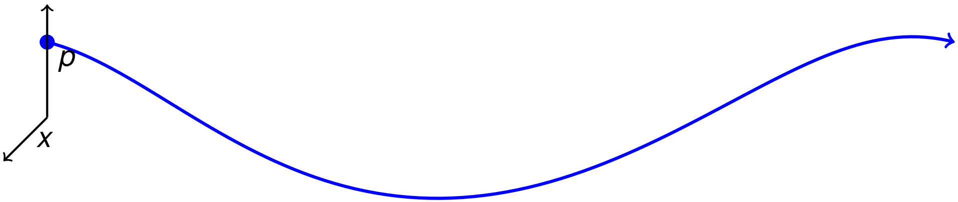

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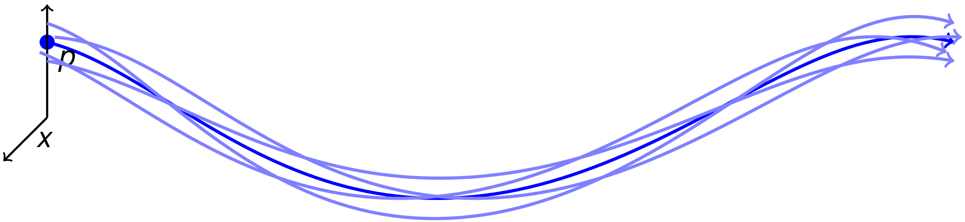

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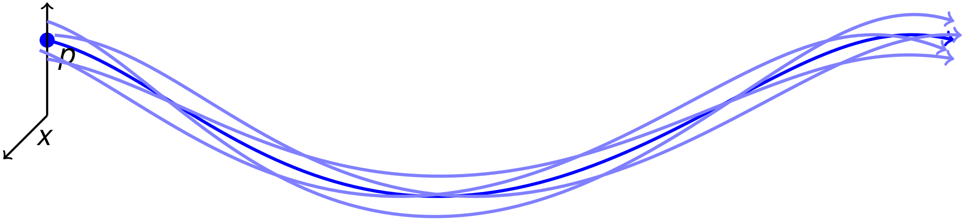

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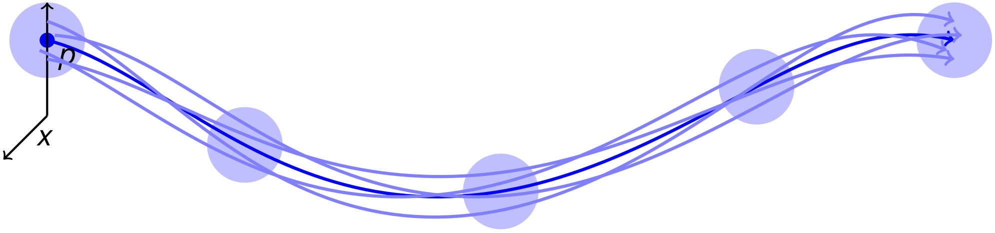
Collisionless gas

Particle density function is constant along particle trajectories in phase space.



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$\Rightarrow$ Particle trajectories are integral curves of X_H.

From particle dynamics to gas dynamics

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$\Rightarrow$ 1-PDF follows Liouville equation: $\mathcal{L}_{X_H}\phi = 0$.

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$\Rightarrow$ Liouville equation becomes $\partial_r \phi = 0$.

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Static spherically symmetric system

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κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^\mu\bar{x}_\mu\right) + \frac{1}{2}e^{\ell Z^\mu\bar{x}_\mu}(g^{\mu\nu} + Z^\mu Z^\nu)\bar{x}_\mu\bar{x}_\nu\,. \tag{8}$$

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$\Rightarrow$ Minimal modification of Schwarzschild spacetime of mass M:

$$a^{-1} = b = c^{-2} = 1 - \frac{2M}{r}\,, \quad d = 0\,. \tag{10}$$

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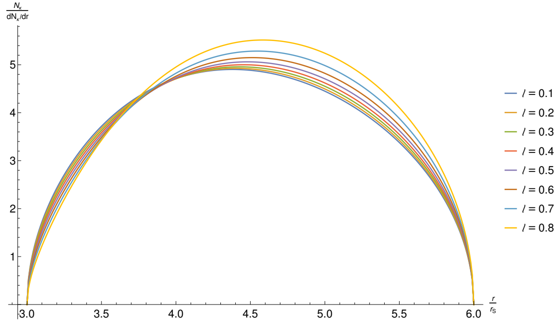

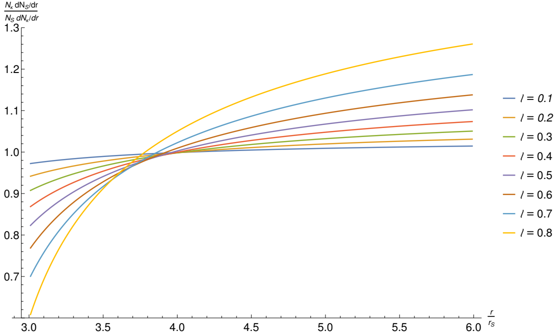

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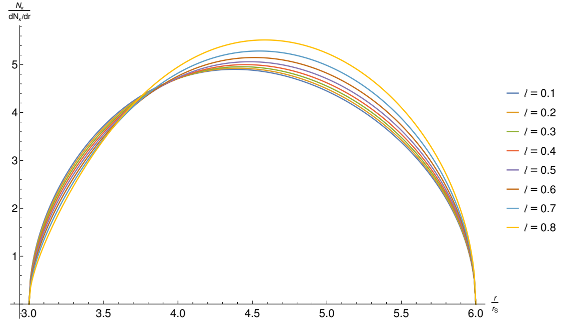

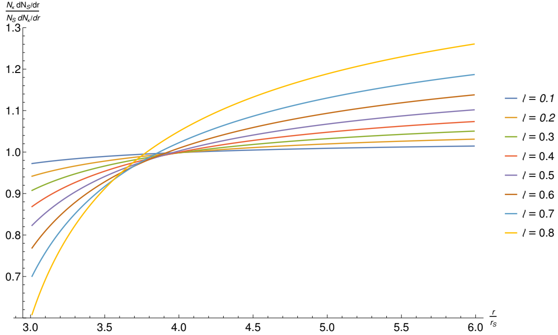

$\Rightarrow$ κ-Poincaré modification shifts particles inward.

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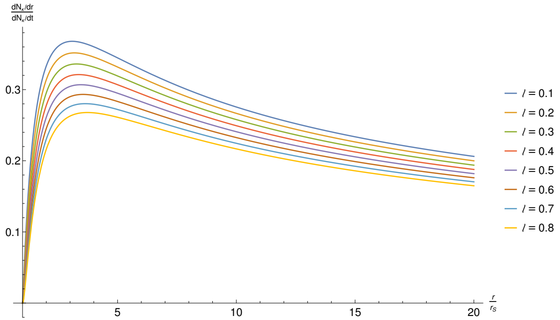

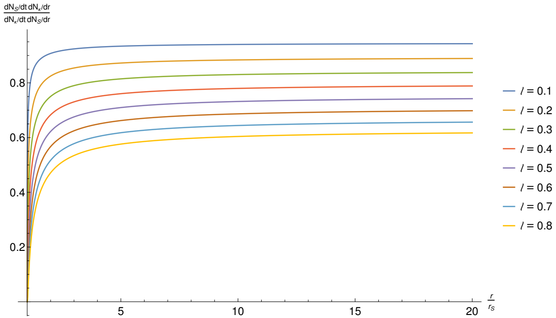

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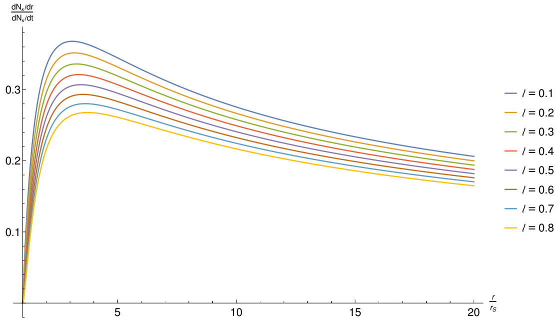

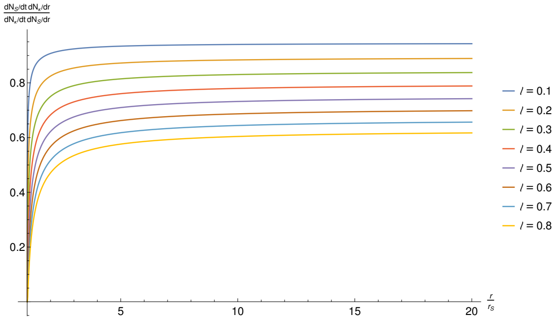

$\Rightarrow$ κ-Poincaré modification decreases particle density.

Conclusion

- Summary:
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- MH, “Kinetic gases in static spherically symmetric modified dispersion relations,” Class. Quant. Grav. **41** (2024) no.1, 015025 [arXiv:2310.01487 [gr-qc]].