Prospects in Finsler gravity and cosmology

Manuel Hohmann

Laboratory of Theoretical Physics, Institute of Physics, University of Tartu
Center of Excellence “Fundamental Universe”

Space Science @ Drop Tower Seminar - 7. November 2025

Overview

Overview

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible (✓).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible (✓).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.
  - Study the phenomenology of quantum corrections.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.
  - Study the phenomenology of quantum corrections.
- How can we study quantum gravity phenomenology?
  - Find physical system which could amplify deviations from general relativity.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible (✓).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.
  - Study the phenomenology of quantum corrections.
- How can we study quantum gravity phenomenology?
  - Find physical system which could amplify deviations from general relativity.
  - Example: study compact system with very strong gravity.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.
  - Study the phenomenology of quantum corrections.
- How can we study quantum gravity phenomenology?
  - Find physical system which could amplify deviations from general relativity.
  - Example: study compact system with very strong gravity.
  - Think of possible observables in the chosen system.

Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.
  - Study the phenomenology of quantum corrections.
- How can we study quantum gravity phenomenology?
  - Find physical system which could amplify deviations from general relativity.
  - Example: study compact system with very strong gravity.
  - Think of possible observables in the chosen system.
  - Calculate how effective quantum gravity influences observables.

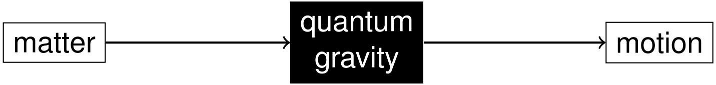
Idea and motivation

- How can we quantize gravity?
  - Use same methods as in QFT $\rightsquigarrow$ doesn't work $\lightning$.
  - Guess a complete theory of quantum gravity $\rightsquigarrow$ hard $\lightning$.
  - Assume gravity is classical $\rightsquigarrow$ leaves unsolved problems $\lightning$.
  - Study effective quantum gravity model phenomenology $\rightsquigarrow$ maybe feasible ($\checkmark$).
- How can we study effective gravity models?
  - Don't care (too much) about fundamental laws of gravity.
  - Assume that general relativity (GR) is almost correct.
  - Think of possible sources of quantum corrections to GR.
  - Study the phenomenology of quantum corrections.
- How can we study quantum gravity phenomenology?
  - Find physical system which could amplify deviations from general relativity.
  - Example: study compact system with very strong gravity.
  - Think of possible observables in the chosen system.
  - Calculate how effective quantum gravity influences observables.

$\Rightarrow$ Here: effective quantum gravity phenomenology with gas dynamics near black holes.

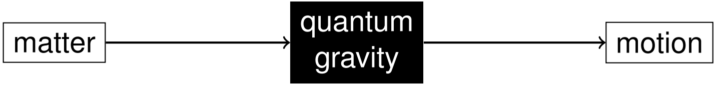
How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:



How does quantum gravity phenomenology work?

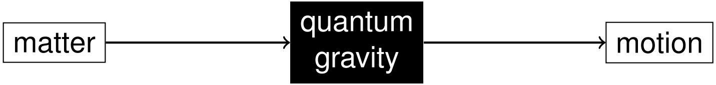
- Basic operating principle of (quantum) gravity theory:



↯ Quantum gravity is a black box!

How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:



↯ Quantum gravity is a black box!

- Quantum gravity must approximate general relativity:

quantum gravity $=$ general relativity $+ \epsilon \cdot$ quantum correction

How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:

matter → quantum gravity → motion

↯ Quantum gravity is a black box!

- Quantum gravity must approximate general relativity:

$$\boxed{\text{quantum gravity}} = \boxed{\text{general relativity}} + \epsilon \cdot \boxed{\text{quantum correction}}$$

⇝ We still have a black box, but it is multiplied by $\epsilon \ll 1$ ⇝ perturbation.

How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:

matter → quantum gravity → motion

↯ Quantum gravity is a black box!

- Quantum gravity must approximate general relativity:

quantum gravity = general relativity + $\epsilon \cdot$ quantum correction

⇝ We still have a black box, but it is multiplied by $\epsilon \ll 1$ ⇝ perturbation.

✓ General relativity is a very simple theory!

How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:

matter → quantum gravity → motion

↯ Quantum gravity is a black box!

- Quantum gravity must approximate general relativity:

quantum gravity = general relativity + $\epsilon \cdot$ quantum correction

⇝ We still have a black box, but it is multiplied by $\epsilon \ll 1$ ⇝ perturbation.

✓ General relativity is a very simple theory!

⇒ We know what is in the white box:

general relativity =

How does quantum gravity phenomenology work?

- Basic operating principle of (quantum) gravity theory:

matter → **quantum gravity** → motion

↯ Quantum gravity is a black box!

- Quantum gravity must approximate general relativity:

quantum gravity = general relativity $+ \epsilon \cdot$ **quantum correction**

⇝ We still have a black box, but it is multiplied by $\epsilon \ll 1$ ⇝ perturbation.

✓ General relativity is a very simple theory!

⇒ We know what is in the white box:

general relativity = 

⇝ Only need to study (all) possible quantum corrections!

Overview

The clock postulate

- Proper time along a curve in Lorentzian spacetime:

$$\tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)}dt\,.$$

The clock postulate

- Proper time along a curve in Lorentzian spacetime:

$$\tau = \int_{t_1}^{t_2} \sqrt{-g_{ab}(x(t))\dot{x}^a(t)\dot{x}^b(t)}dt \,.$$

- Finsler geometry: use a more general length functional:

$$\tau = \int_{t_1}^{t_2} F(x(t), \dot{x}(t))dt \,.$$

- Finsler function $F : TM \to \mathbb{R}^+$.
- Parametrization invariance requires homogeneity:

$$F(x, \lambda y) = \lambda F(x, y) \quad \forall \lambda > 0 \,.$$

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

⇒ Finsler metric with Lorentz signature:

$$g^F_{ab}(x,y) = \frac{1}{2}\frac{\partial}{\partial y^a}\frac{\partial}{\partial y^b}F^2(x,y)\,.$$

⇒ Notion of timelike, lightlike, spacelike tangent vectors.

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

$\Rightarrow$ Finsler metric with Lorentz signature:

$$g^F_{ab}(x,y) = \frac{1}{2}\frac{\partial}{\partial y^a}\frac{\partial}{\partial y^b}F^2(x,y)\,.$$

$\Rightarrow$ Notion of timelike, lightlike, spacelike tangent vectors.

- Unit vectors $y \in T_xM$ defined by

$$F^2(x,y) = g^F_{ab}(x,y)y^a y^b = 1\,.$$

$\Rightarrow$ Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.

Finsler spacetimes

- Finsler geometries suitable for spacetimes exist. [C. Pfeifer, M. Wohlfarth '11]

⇒ Finsler metric with Lorentz signature:

$$g^F_{ab}(x,y) = \frac{1}{2}\frac{\partial}{\partial y^a}\frac{\partial}{\partial y^b}F^2(x,y)\,.$$

⇒ Notion of timelike, lightlike, spacelike tangent vectors.

- Unit vectors $y \in T_xM$ defined by

$$F^2(x,y) = g^F_{ab}(x,y)y^a y^b = 1\,.$$

⇒ Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.

- Ω_x contains a closed connected component $S_x \subseteq \Omega_x$.

⇝ Causality: S_x corresponds to physical observers.

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]$$

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b \left[g^{F\,ac}(y^d \partial_d \bar{\partial}_c F^2 - \partial_c F^2)\right]$$

⇒ Split of the tangent and cotangent bundles:

- Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a \bar{\partial}_b\,, \quad \bar{\partial}_a$$

- Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

$$dx^a\,, \quad \delta y^a = dy^a + N^a{}_b dx^b$$

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]$$

⇒ Split of the tangent and cotangent bundles:

- Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a\bar{\partial}_b\,, \quad \bar{\partial}_a$$

- Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

$$dx^a\,, \quad \delta y^a = dy^a + N^a{}_b dx^b$$

- Sasaki metric:

$$G = -g^F_{ab}\,dx^a \otimes dx^b - \frac{g^F_{ab}}{F^2}\,\delta y^a \otimes \delta y^b$$

Geometry on the tangent bundle

- Cartan non-linear connection:

$$N^a{}_b = \frac{1}{4}\bar{\partial}_b\left[g^{F\,ac}(y^d\partial_d\bar{\partial}_c F^2 - \partial_c F^2)\right]$$

⇒ Split of the tangent and cotangent bundles:
  - Tangent bundle: $TTM = HTM \oplus VTM$

$$\delta_a = \partial_a - N^b{}_a\bar{\partial}_b\,, \quad \bar{\partial}_a$$

  - Cotangent bundle: $T^*TM = H^*TM \oplus V^*TM$

$$dx^a\,, \quad \delta y^a = dy^a + N^a{}_b dx^b$$

- Sasaki metric:

$$G = -g^F_{ab}\,dx^a \otimes dx^b - \frac{g^F_{ab}}{F^2}\,\delta y^a \otimes \delta y^b$$

- Geodesic spray:

$$\mathbf{S} = y^a\delta_a$$

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
  - Physical observers correspond to $S_x \subseteq \Omega_x$.
- Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM\,.$$

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
  - Physical observers correspond to $S_x \subseteq \Omega_x$.
- Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM\,.$$

- Sasaki metric $\tilde{G}$ on O given by pullback of G to O.
- Volume form Σ of Sasaki metric $\tilde{G}$.
- Geodesic spray **S** restricts to Reeb vector field **r** on O.

Geometry on observer space

- Recall from the definition of Finsler spacetimes:
  - Set $\Omega_x \subset T_xM$ of unit timelike vectors at $x \in M$.
  - Physical observers correspond to $S_x \subseteq \Omega_x$.
- Definition of observer space:

$$O = \bigcup_{x \in M} S_x \subset TM\,.$$

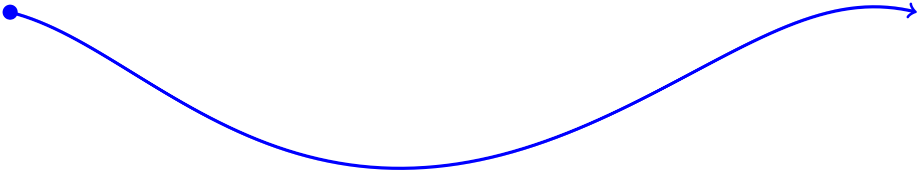
- Sasaki metric $\tilde{G}$ on O given by pullback of G to O.
- Volume form Σ of Sasaki metric $\tilde{G}$.
- Geodesic spray $\mathbf{S}$ restricts to Reeb vector field $\mathbf{r}$ on O.
- Geodesic hypersurface measure $\omega = \iota_{\mathbf{r}}\Sigma$.
- Note that $\mathcal{L}_{\mathbf{r}}\Sigma = 0$ and $d\omega = 0$.

Kinetic gas to probe quantum gravity

- Gas is constituted by particles of equal mass.

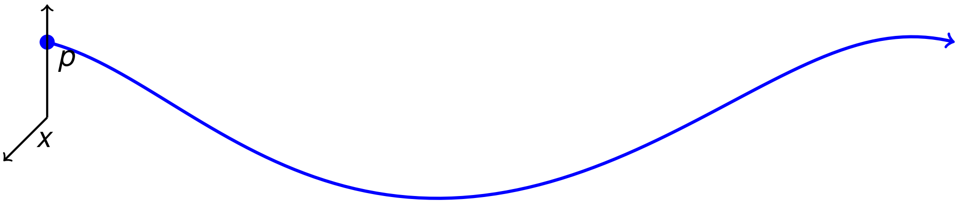
Kinetic gas to probe quantum gravity

- Gas is constituted by particles of equal mass.
- Particle trajectories follow (relativistic) Hamiltonian dynamics.



Kinetic gas to probe quantum gravity

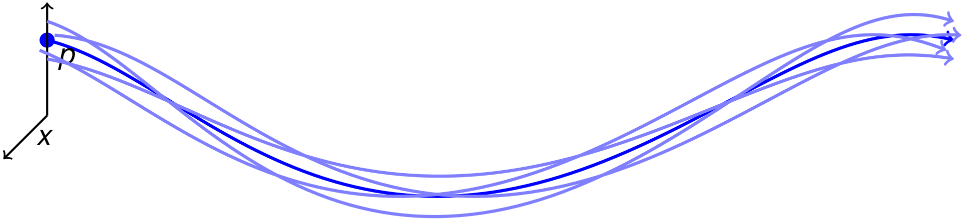
- Gas is constituted by particles of equal mass.
- Particle trajectories follow (relativistic) Hamiltonian dynamics.
- Each particle is described by (spacetime) position and four-momentum.



Kinetic gas to probe quantum gravity

- Gas is constituted by particles of equal mass.
- Particle trajectories follow (relativistic) Hamiltonian dynamics.
- Each particle is described by (spacetime) position and four-momentum.

⇝ Kinetic gas is density distribution in 8-dimensional position-momentum phase space.

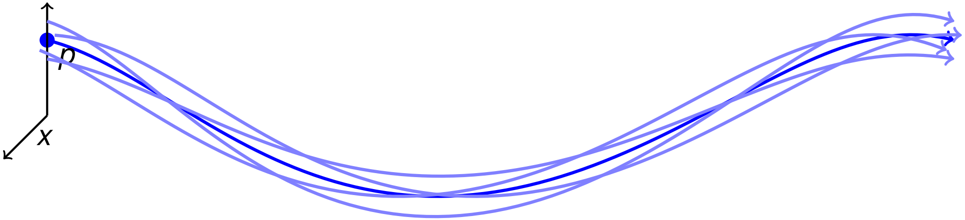


Kinetic gas to probe quantum gravity

- Gas is constituted by particles of equal mass.
- Particle trajectories follow (relativistic) Hamiltonian dynamics.
- Each particle is described by (spacetime) position and four-momentum.

⇝ Kinetic gas is density distribution in 8-dimensional position-momentum phase space.

⇒ Gas dynamics follows from Hamiltonian particle dynamics.



Kinetic gas to probe quantum gravity

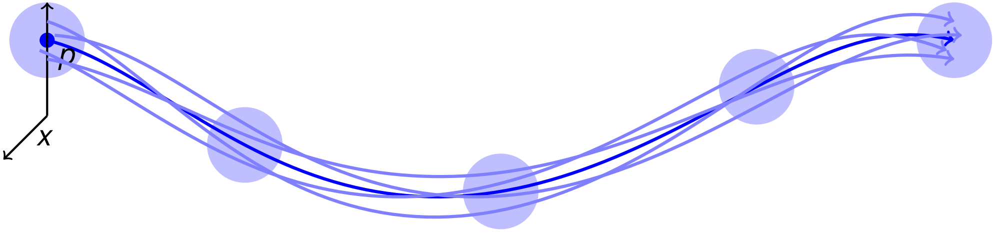
- Gas is constituted by particles of equal mass.
- Particle trajectories follow (relativistic) Hamiltonian dynamics.
- Each particle is described by (spacetime) position and four-momentum.

$\rightsquigarrow$ Kinetic gas is density distribution in 8-dimensional position-momentum phase space.

$\Rightarrow$ Gas dynamics follows from Hamiltonian particle dynamics.

Collisionless gas

Particle density function is constant along particle trajectories in phase space.



Finsler formulation of relativistic particle dynamics

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

Finsler formulation of relativistic particle dynamics

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

- Canonical lift of curve to tangent bundle TM:

$$x\,, \quad y = \dot{x}\,.$$

- Lift of geodesic equation:

$$\dot{x}^a = y^a\,, \quad \dot{y}^a = -N^a{}_b(x, y)y^b\,.$$

Finsler formulation of relativistic particle dynamics

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0 .$$

- Canonical lift of curve to tangent bundle TM:

$$x , \quad y = \dot{x} .$$

- Lift of geodesic equation:

$$\dot{x}^a = y^a , \quad \dot{y}^a = -N^a{}_b(x, y)y^b .$$

⇒ Solutions are integral curves of vector field:

$$y^a\partial_a - y^b N^a{}_b\bar{\partial}_a = y^a\delta_a = \mathbf{S} .$$

Finsler formulation of relativistic particle dynamics

- Dynamics of fluids depends on geodesic equation.
- Geodesic equation for curve $x(\tau)$ on spacetime M:

$$\ddot{x}^a + N^a{}_b(x, \dot{x})\dot{x}^b = 0\,.$$

- Canonical lift of curve to tangent bundle TM:

$$x\,, \quad y = \dot{x}\,.$$

- Lift of geodesic equation:

$$\dot{x}^a = y^a\,, \quad \dot{y}^a = -N^a{}_b(x, y)y^b\,.$$

⇒ Solutions are integral curves of vector field:

$$y^a\partial_a - y^b N^a{}_b\bar{\partial}_a = y^a\delta_a = \mathbf{S}\,.$$

- Tangent vectors are future unit timelike: $(x, y) \in O$.

⇒ Particle trajectories are piecewise integral curves of **r** on O.

Hamiltonian formulation of relativistic particle dynamics

- Model particle dynamics on cotangent bundle T^*M with coordinates $(x^a, \bar{x}_a)$.

Hamiltonian formulation of relativistic particle dynamics

- Model particle dynamics on cotangent bundle T^*M with coordinates $(x^a, \bar{x}_a)$.
- (Modified) dispersion relation: mass shell condition for Hamiltonian:

$$-\frac{m^2}{2} = H(x^a, \bar{x}_a)\,. \tag{1}$$

Hamiltonian formulation of relativistic particle dynamics

- Model particle dynamics on cotangent bundle T^*M with coordinates $(x^a, \bar{x}_a)$.
- (Modified) dispersion relation: mass shell condition for Hamiltonian:

$$-\frac{m^2}{2} = H(x^a, \bar{x}_a)\,. \tag{1}$$

- Particle trajectories derived from Hamilton's equations of motion:

$$\dot{x}^a = \bar{\partial}^a H\,, \quad \dot{\bar{x}}^a = -\partial_a H\,. \tag{2}$$

Hamiltonian formulation of relativistic particle dynamics

- Model particle dynamics on cotangent bundle T^*M with coordinates $(x^a, \bar{x}_a)$.
- (Modified) dispersion relation: mass shell condition for Hamiltonian:

$$-\frac{m^2}{2} = H(x^a, \bar{x}_a)\,. \tag{1}$$

- Particle trajectories derived from Hamilton's equations of motion:

$$\dot{x}^a = \bar{\partial}^a H\,, \quad \dot{\bar{x}}^a = -\partial_a H\,. \tag{2}$$

- Canonical cotangent bundle geometry: symplectic form $\omega \in \Omega^2(T^*M)$ as

$$\theta = \bar{x}_a \mathrm{d}x^a\,, \quad \omega = \mathrm{d}\theta = \mathrm{d}\bar{x}_a \wedge \mathrm{d}x^a\,. \tag{3}$$

Hamiltonian formulation of relativistic particle dynamics

- Model particle dynamics on cotangent bundle T^*M with coordinates $(x^a, \bar{x}_a)$.
- (Modified) dispersion relation: mass shell condition for Hamiltonian:

$$-\frac{m^2}{2} = H(x^a, \bar{x}_a)\,. \tag{1}$$

- Particle trajectories derived from Hamilton's equations of motion:

$$\dot{x}^a = \bar{\partial}^a H\,, \quad \dot{\bar{x}}^a = -\partial_a H\,. \tag{2}$$

- Canonical cotangent bundle geometry: symplectic form $\omega \in \Omega^2(T^*M)$ as

$$\theta = \bar{x}_a \mathrm{d}x^a\,, \quad \omega = \mathrm{d}\theta = \mathrm{d}\bar{x}_a \wedge \mathrm{d}x^a\,. \tag{3}$$

- Hamiltonian vector field X_H on T^*M: unique solution of

$$\iota_{X_H}\omega = -\mathrm{d}H\,. \tag{4}$$

Hamiltonian formulation of relativistic particle dynamics

- Model particle dynamics on cotangent bundle T^*M with coordinates $(x^a, \bar{x}_a)$.
- (Modified) dispersion relation: mass shell condition for Hamiltonian:

$$-\frac{m^2}{2} = H(x^a, \bar{x}_a)\,. \tag{1}$$

- Particle trajectories derived from Hamilton's equations of motion:

$$\dot{x}^a = \bar{\partial}^a H\,, \quad \dot{\bar{x}}^a = -\partial_a H\,. \tag{2}$$

- Canonical cotangent bundle geometry: symplectic form $\omega \in \Omega^2(T^*M)$ as

$$\theta = \bar{x}_a \mathrm{d}x^a\,, \quad \omega = \mathrm{d}\theta = \mathrm{d}\bar{x}_a \wedge \mathrm{d}x^a\,. \tag{3}$$

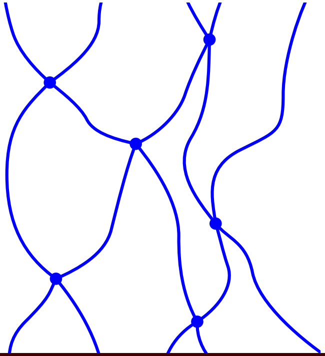
- Hamiltonian vector field X_H on T^*M: unique solution of

$$\iota_{X_H}\omega = -\mathrm{d}H\,. \tag{4}$$

⇒ Particle trajectories are integral curves of X_H.

Definition of kinetic gas

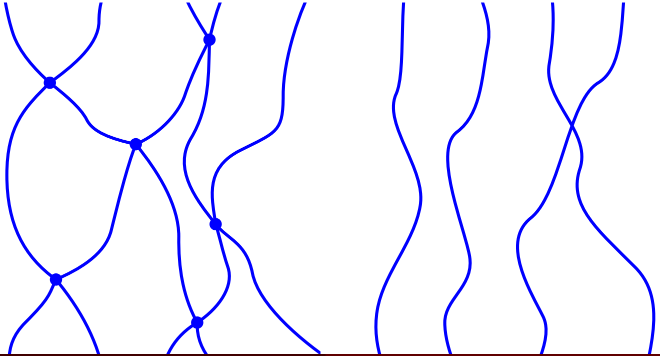
- Single-component gas:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.



Definition of kinetic gas

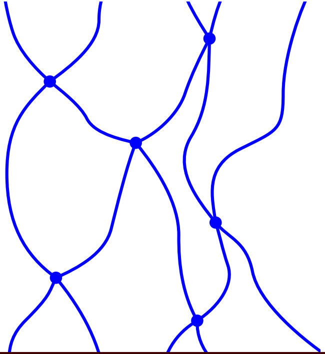
- Single-component gas:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.
- Collisionless gas:
  - Particles do not interact with other particles.

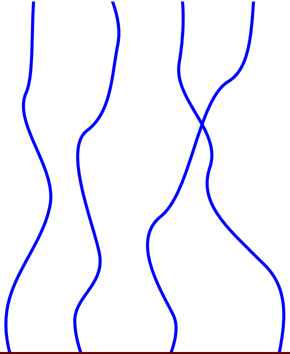
  $\Rightarrow$ Particles follow geodesics.

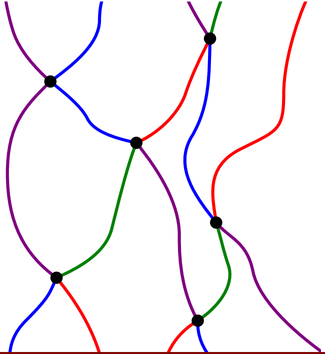


Definition of kinetic gas

- Single-component gas:
  - Constituted by classical, relativistic particles.
  - Particles have equal properties (mass, charge, . . .).
  - Particles follow piecewise geodesic curves.
  - Endpoints of geodesics are interactions with other particles.
- Collisionless gas:
  - Particles do not interact with other particles.
  - ⇒ Particles follow geodesics.
- Multi-component gas: multiple types of particles.







One-particle distribution function

- Kinetic gas described by density in velocity space:
  - Consider space O of physical (unit, timelike, future pointing) four-velocities.
  - Consider density on physical velocity space.

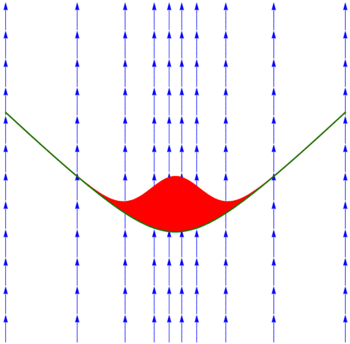
One-particle distribution function

- Kinetic gas described by density in velocity space:
  - Consider space O of physical (unit, timelike, future pointing) four-velocities.
  - Consider density on physical velocity space.
- Define one-particle distribution function $\phi : O \to \mathbb{R}^+$ such that:

For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi \Omega$$

# of particle trajectories through σ.



  -
  - Counting of particle trajectories respects hypersurface orientation.

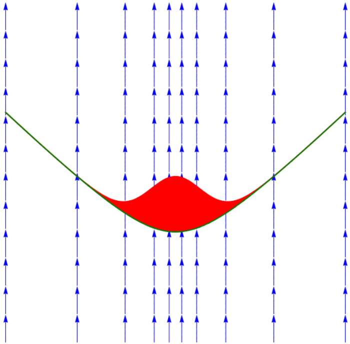
One-particle distribution function

- Kinetic gas described by density in velocity space:
  - Consider space O of physical (unit, timelike, future pointing) four-velocities.
  - Consider density on physical velocity space.
- Define one-particle distribution function $\phi : O \to \mathbb{R}^+$ such that:

For every hypersurface $\sigma \subset O$,

$$N[\sigma] = \int_\sigma \phi\Omega$$

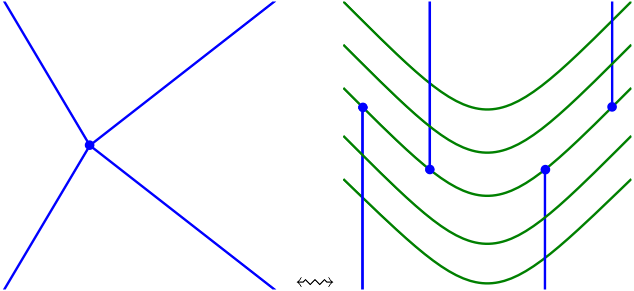
# of particle trajectories through σ.



  -
  - Counting of particle trajectories respects hypersurface orientation.
- For multi-component fluids: ϕ_i for each component i.

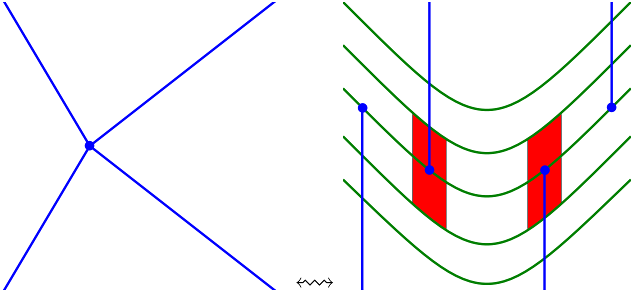
Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



- For any open set $V \in O$,

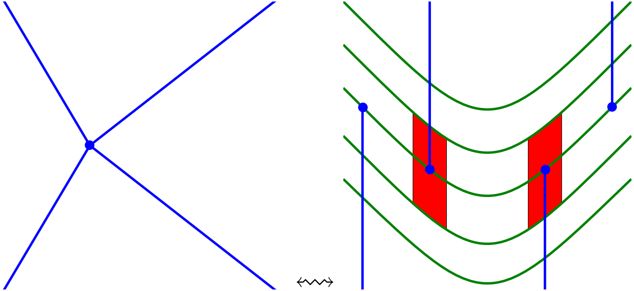
$$\int_{\partial V} \phi\Omega = \int_V d(\phi\Omega) = \int_V \mathcal{L}_{\mathbf{r}}\phi\Sigma$$

  # of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}}\phi$.

Collisions & the Liouville equation

- Collision in spacetime $\leftrightsquigarrow$ interruption in observer space.



- For any open set $V \in O$,

$$\int_{\partial V} \phi\Omega = \int_V d(\phi\Omega) = \int_V \mathcal{L}_{\mathbf{r}}\phi\Sigma$$

  # of outbound trajectories - # of inbound trajectories.

$\Rightarrow$ Collision density measured by $\mathcal{L}_{\mathbf{r}}\phi$.

- Collisionless fluid: trajectories have no endpoints, $\mathcal{L}_{\mathbf{r}}\phi = 0$.

$\Rightarrow$ Simple, first order equation of motion for collisionless fluid.

$\Rightarrow$ ϕ is constant along integral curves of $\mathbf{r}$.

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}\bar{x}_0 \wedge \mathrm{d}\bar{x}_1 \wedge \mathrm{d}\bar{x}_2 \wedge \mathrm{d}\bar{x}_3 \,. \tag{5}$$

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\bar{x}_0 \wedge d\bar{x}_1 \wedge d\bar{x}_2 \wedge d\bar{x}_3 \,. \qquad (5)$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}\bar{x}_0 \wedge \mathrm{d}\bar{x}_1 \wedge \mathrm{d}\bar{x}_2 \wedge \mathrm{d}\bar{x}_3\,. \tag{5}$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.
- One particle distribution function $\phi : T^*M \to \mathbb{R}^+$:

$$N[\sigma] = \int_\sigma \phi\Omega\,. \tag{6}$$

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}\bar{x}_0 \wedge \mathrm{d}\bar{x}_1 \wedge \mathrm{d}\bar{x}_2 \wedge \mathrm{d}\bar{x}_3 \,. \tag{5}$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.
- One particle distribution function $\phi : T^*M \to \mathbb{R}^+$:

$$N[\sigma] = \int_\sigma \phi \Omega \,. \tag{6}$$

  - σ: hypersurface in T^*M which is transverse to X_H.

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\bar{x}_0 \wedge d\bar{x}_1 \wedge d\bar{x}_2 \wedge d\bar{x}_3 \,. \tag{5}$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.
- One particle distribution function $\phi : T^*M \to \mathbb{R}^+$:

$$N[\sigma] = \int_\sigma \phi\Omega \,. \tag{6}$$

  - σ: hypersurface in T^*M which is transverse to X_H.
  - $N[\sigma]$: number of particle trajectories through σ.

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\bar{x}_0 \wedge d\bar{x}_1 \wedge d\bar{x}_2 \wedge d\bar{x}_3 \,. \tag{5}$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.
- One particle distribution function $\phi : T^*M \to \mathbb{R}^+$:

$$N[\sigma] = \int_\sigma \phi \Omega \,. \tag{6}$$

  - σ: hypersurface in T^*M which is transverse to X_H.
  - $N[\sigma]$: number of particle trajectories through σ.
  - $\Omega = \iota_{X_H}\Sigma$: particle measure .

Hamiltonian formulation of gas dynamics

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\bar{x}_0 \wedge d\bar{x}_1 \wedge d\bar{x}_2 \wedge d\bar{x}_3 \,. \tag{5}$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.
- One particle distribution function $\phi : T^*M \to \mathbb{R}^+$:

$$N[\sigma] = \int_\sigma \phi\Omega \,. \tag{6}$$

  - σ: hypersurface in T^*M which is transverse to X_H.
  - $N[\sigma]$: number of particle trajectories through σ.
  - $\Omega = \iota_{X_H}\Sigma$: particle measure .
- Collisionless gas: particles follow Hamilton's equations of motion, no interactions.

- Introduce symplectic volume form:

$$\Sigma = \frac{1}{4!}\omega \wedge \omega \wedge \omega \wedge \omega = \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3 \wedge \mathrm{d}\bar{x}_0 \wedge \mathrm{d}\bar{x}_1 \wedge \mathrm{d}\bar{x}_2 \wedge \mathrm{d}\bar{x}_3 \,. \tag{5}$$

- Kinetic gas: particles of equal mass described by phase space trajectories on T^*M.
- One particle distribution function $\phi : T^*M \to \mathbb{R}^+$:

$$N[\sigma] = \int_\sigma \phi\Omega \,. \tag{6}$$

  - σ: hypersurface in T^*M which is transverse to X_H.
  - $N[\sigma]$: number of particle trajectories through σ.
  - $\Omega = \iota_{X_H}\Sigma$: particle measure .
- Collisionless gas: particles follow Hamilton's equations of motion, no interactions.

$\Rightarrow$ 1-PDF follows Liouville equation: $\mathcal{L}_{X_H}\phi = 0$.

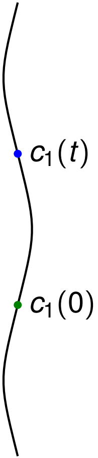
Action of a kinetic gas

Action for a single point particle:

$$S = m \int_0^t (F \circ c_1)(\tau)\, d\tau\,.$$

Assume arc length parameter τ:

$$S = mt\,.$$



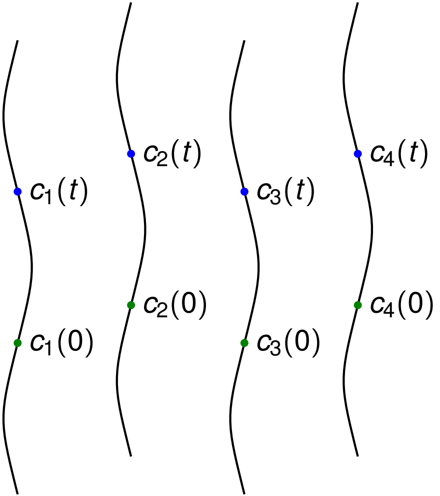
Action of a kinetic gas

Action for P point particles:

$$S_{\text{gas}} = m \sum_{i=1}^{P} \int_0^t (F \circ c_i)(\tau)\, d\tau\,.$$

Assume arc length parameter τ:

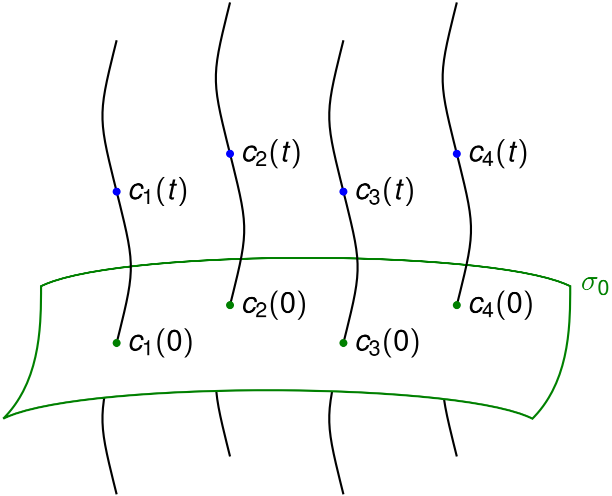
$$S_{\text{gas}} = Pmt\,.$$



Action of a kinetic gas

- Hypersurface of starting points:

$$c_i(0) \in \sigma_0\,.$$



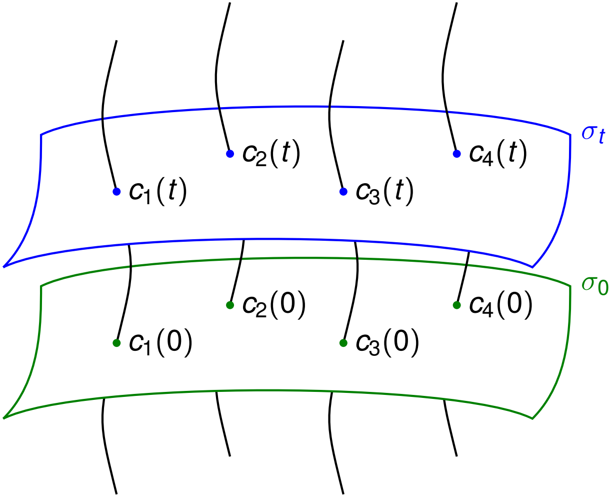
Action of a kinetic gas

- Hypersurface of starting points:

$$c_i(0) \in \sigma_0\,.$$

- Hypersurface of end points:

$$c_i(t) \in \sigma_t\,.$$



Action of a kinetic gas

- Hypersurface of starting points:

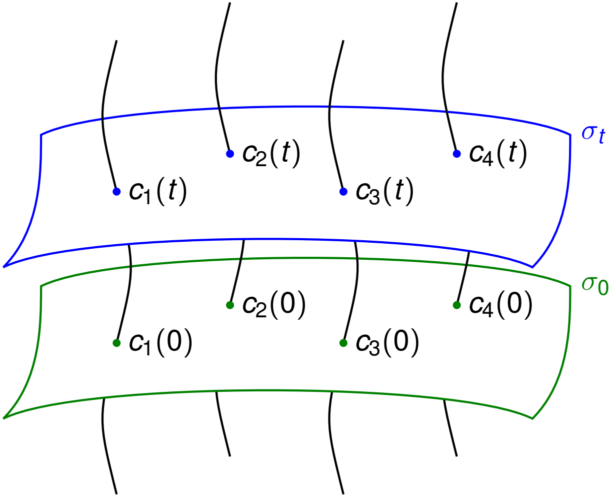
$$c_i(0) \in \sigma_0\,.$$

- Hypersurface of end points:

$$c_i(t) \in \sigma_t\,.$$

- Number of particle trajectories:

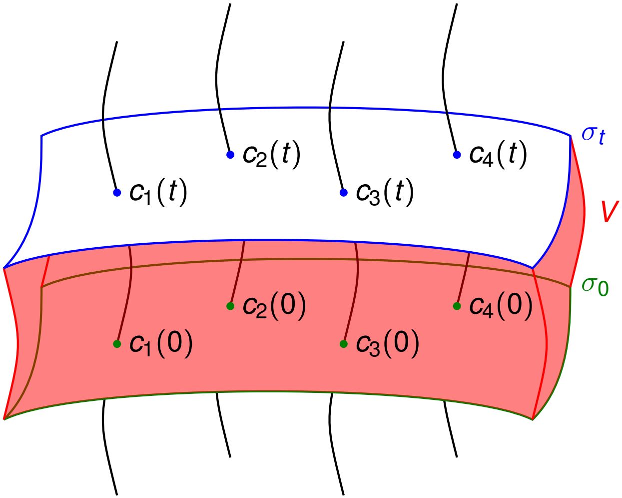
$$P = N[\sigma_\tau] = \int_{\sigma_\tau} \phi\Omega\,.$$



Action of a kinetic gas

- Consider volume

$$V = \bigcup_{\tau=0}^{t} \sigma_\tau \,.$$



Action of a kinetic gas

- Consider volume

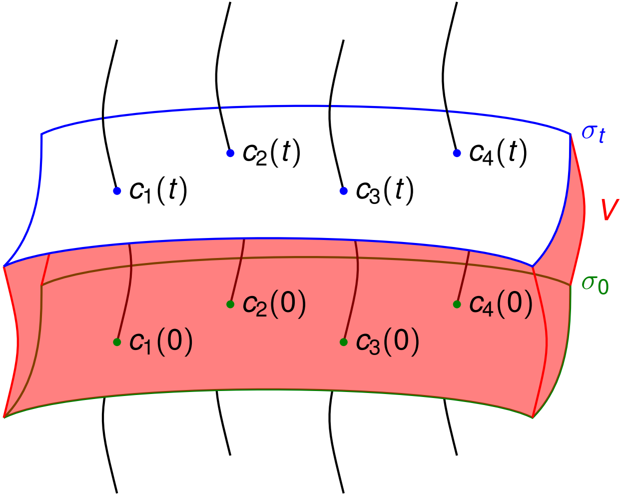
$$V = \bigcup_{\tau=0}^{t} \sigma_\tau \,.$$

- Recall particle action integral:

$$S_{\text{gas}} = Pmt = m\int_0^t \left(\int_{\sigma_\tau} \phi\Omega\right) d\tau$$
$$= m\int_V \phi\Omega \wedge \omega$$
$$= m\int_V \phi\Sigma \,.$$

Defined through 1-PDF ϕ

[MH, Pfeifer, Voicu '19].



Action of a kinetic gas

- Consider volume

$$V = \bigcup_{\tau=0}^{t} \sigma_\tau \, .$$

- Recall particle action integral:

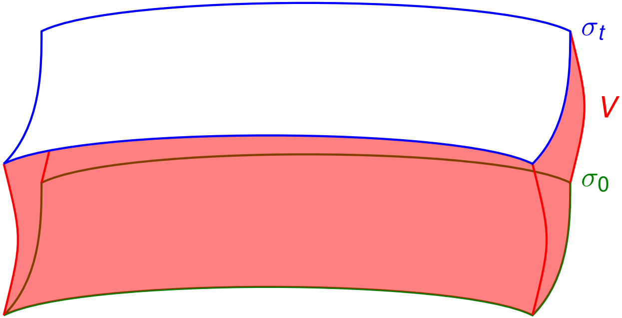
$$S_{\text{gas}} = Pmt = m \int_0^t \left(\int_{\sigma_\tau} \phi \Omega \right) d\tau$$

$$= m \int_V \phi \Omega \wedge \omega$$

$$= m \int_V \phi \Sigma \, .$$

Defined through 1-PDF ϕ

[MH, Pfeifer, Voicu '19].



⇒ Forget particle trajectories!

Gravitational action

- Gravitational part of the action:

$$S_{\text{grav}} = \frac{1}{2\kappa^2} \int_V R_0 \Sigma \,.$$

Gravitational action

- Gravitational part of the action:

$$S_{\mathrm{grav}} = \frac{1}{2\kappa^2} \int_V R_0 \Sigma \,.$$

- Finsler Ricci scalar $R_0 = L^{-1} R^a{}_{ab} y^b$ from curvature of non-linear connection:

$$R^a{}_{bc} \bar{\partial}_a = (\delta_b N^a{}_c - \delta_c N^a{}_b) \bar{\partial}_a = [\delta_b, \delta_c] \,.$$

Gravitational action

- Gravitational part of the action:

$$S_{\text{grav}} = \frac{1}{2\kappa^2} \int_V R_0 \Sigma \,.$$

- Finsler Ricci scalar $R_0 = L^{-1} R^a{}_{ab} y^b$ from curvature of non-linear connection:

$$R^a{}_{bc} \bar{\partial}_a = (\delta_b N^a{}_c - \delta_c N^a{}_b) \bar{\partial}_a = [\delta_b, \delta_c] \,.$$

! Unique action obtained from variational completion of Rutz equation [MH, Pfeifer, Voicu '18].

Gravitational action

- Gravitational part of the action:

$$S_{\text{grav}} = \frac{1}{2\kappa^2} \int_V R_0 \Sigma \,.$$

- Finsler Ricci scalar $R_0 = L^{-1} R^a{}_{ab} y^b$ from curvature of non-linear connection:

$$R^a{}_{bc} \bar{\partial}_a = (\delta_b N^a{}_c - \delta_c N^a{}_b) \bar{\partial}_a = [\delta_b, \delta_c] \,.$$

! Unique action obtained from variational completion of Rutz equation [MH, Pfeifer, Voicu '18].

⇒ Reduces to Einstein-Hilbert action for metric geometry.

Variation and field equations

- Variation of the kinetic gas action:

$$\delta_F S_{\text{gas}} = \int_V \phi \frac{\delta F}{F} \Sigma \,.$$

Variation and field equations

- Variation of the kinetic gas action:

$$\delta_F S_{\text{gas}} = \int_V \phi \frac{\delta F}{F} \Sigma\,.$$

- Variation of the Finsler gravity action:

$$\delta_F S_{\text{grav}} = 2 \int_V \left[\frac{1}{2} g^{F\,ab} \bar{\partial}_a \bar{\partial}_b (F^2 R_0) - 3 R_0 - g^{F\,ab} (\nabla_{\delta_a} P_b - P_a P_b + \bar{\partial}_a (\nabla P_b))\right] \frac{\delta F}{F} \Sigma\,.$$

Variation and field equations

- Variation of the kinetic gas action:

$$\delta_F S_{\text{gas}} = \int_V \phi \frac{\delta F}{F} \Sigma \,.$$

- Variation of the Finsler gravity action:

$$\delta_F S_{\text{grav}} = 2 \int_V \left[\frac{1}{2} g^{F\,ab} \bar{\partial}_a \bar{\partial}_b (F^2 R_0) - 3 R_0 - g^{F\,ab} (\nabla_{\delta_a} P_b - P_a P_b + \bar{\partial}_a (\nabla P_b)) \right] \frac{\delta F}{F} \Sigma \,.$$

- Landsberg tensor measures deviation from metric geometry:

$$P^a{}_{bc} = \bar{\partial}_c N^a{}_b - \Gamma^a{}_{cb} \,, \quad P_a = P^b{}_{ba} \,.$$

Variation and field equations

- Variation of the kinetic gas action:

$$\delta_F S_{\text{gas}} = \int_V \phi \frac{\delta F}{F} \Sigma \,.$$

- Variation of the Finsler gravity action:

$$\delta_F S_{\text{grav}} = 2\int_V \left[\frac{1}{2} g^{F\,ab} \bar{\partial}_a \bar{\partial}_b (F^2 R_0) - 3R_0 - g^{F\,ab} (\nabla_{\delta_a} P_b - P_a P_b + \bar{\partial}_a (\nabla P_b))\right] \frac{\delta F}{F} \Sigma \,.$$

- Landsberg tensor measures deviation from metric geometry:

$$P^a{}_{bc} = \bar{\partial}_c N^a{}_b - \Gamma^a{}_{cb} \,, \quad P_a = P^b{}_{ba} \,.$$

⇒ Gravitational field equations with kinetic gas matter [MH, Pfeifer, Voicu '19]:

$$\frac{1}{2} g^{F\,ab} \bar{\partial}_a \bar{\partial}_b (F^2 R_0) - 3R_0 - g^{F\,ab} (\nabla_{\delta_a} P_b - P_a P_b + \bar{\partial}_a (\nabla P_b)) = -\kappa^2 \phi$$

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

$\Rightarrow$ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

$\Rightarrow$ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

$\rightsquigarrow$ Introduce new coordinates $(t, r, \Theta, \Phi, \Psi, E, P, L)$ such that:

- Θ, Φ, E, L are constant along trajectories (Noether symmetries).
- Hamiltonian and 1-PDF depend only on r, P, E, L.

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

$\Rightarrow$ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

$\rightsquigarrow$ Introduce new coordinates $(t, r, \Theta, \Phi, \Psi, E, P, L)$ such that:

  - Θ, Φ, E, L are constant along trajectories (Noether symmetries).
  - Hamiltonian and 1-PDF depend only on r, P, E, L.

- Also Hamiltonian is constant of motion: $X_H H = 0$.

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

$\Rightarrow$ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

$\rightsquigarrow$ Introduce new coordinates $(t, r, \Theta, \Phi, \Psi, E, P, L)$ such that:

- Θ, Φ, E, L are constant along trajectories (Noether symmetries).
- Hamiltonian and 1-PDF depend only on r, P, E, L.

- Also Hamiltonian is constant of motion: $X_H H = 0$.

$\rightsquigarrow$ Replace P by H in new coordinates $(t, r, \Theta, \Phi, \Psi, E, H, L)$ such that:

- Θ, Φ, E, L, H are constant along trajectories.
- 1-PDF depends only on r, E, L, H.

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

$\Rightarrow$ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

$\rightsquigarrow$ Introduce new coordinates $(t, r, \Theta, \Phi, \Psi, E, P, L)$ such that:

- Θ, Φ, E, L are constant along trajectories (Noether symmetries).
- Hamiltonian and 1-PDF depend only on r, P, E, L.

- Also Hamiltonian is constant of motion: $X_H H = 0$.

$\rightsquigarrow$ Replace P by H in new coordinates $(t, r, \Theta, \Phi, \Psi, E, H, L)$ such that:

- Θ, Φ, E, L, H are constant along trajectories.
- 1-PDF depends only on r, E, L, H.

$\Rightarrow$ Liouville equation becomes $\partial_r \phi = 0$.

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

⇒ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

⇝ Introduce new coordinates $(t, r, \Theta, \Phi, \Psi, E, P, L)$ such that:
  - Θ, Φ, E, L are constant along trajectories (Noether symmetries).
  - Hamiltonian and 1-PDF depend only on r, P, E, L.

- Also Hamiltonian is constant of motion: $X_H H = 0$.

⇝ Replace P by H in new coordinates $(t, r, \Theta, \Phi, \Psi, E, H, L)$ such that:
  - Θ, Φ, E, L, H are constant along trajectories.
  - 1-PDF depends only on r, E, L, H.

⇒ Liouville equation becomes $\partial_r \phi = 0$.

⇒ Most general solution to static spherically symmetric gas: $\phi = \phi(E, L, H)$.

Static spherically symmetric system

- Symmetry generated by vector fields $(X_I) = (X_0, X_1, X_2, X_3)$.

$\Rightarrow$ Hamiltonian and 1-PDF invariant under complete lift: $\mathcal{L}_{\hat{X}_I} H = \mathcal{L}_{\hat{X}_I} \phi = 0$ with

$$\hat{X} = X^a \partial_a - \bar{x}_b \partial_a X^b \bar{\partial}^a \,. \tag{7}$$

$\rightsquigarrow$ Introduce new coordinates $(t, r, \Theta, \Phi, \Psi, E, P, L)$ such that:

  - Θ, Φ, E, L are constant along trajectories (Noether symmetries).
  - Hamiltonian and 1-PDF depend only on r, P, E, L.

- Also Hamiltonian is constant of motion: $X_H H = 0$.

$\rightsquigarrow$ Replace P by H in new coordinates $(t, r, \Theta, \Phi, \Psi, E, H, L)$ such that:

  - Θ, Φ, E, L, H are constant along trajectories.
  - 1-PDF depends only on r, E, L, H.

$\Rightarrow$ Liouville equation becomes $\partial_r \phi = 0$.

$\Rightarrow$ Most general solution to static spherically symmetric gas: $\phi = \phi(E, L, H)$.

$\rightsquigarrow$ Consider gas $\phi \sim \delta(E)\delta(L)\delta(H)$ of identical energy, angular momentum, mass.

Quantum corrected black hole

- Schwarzschild black hole:
  - Spherically symmetric spacetime.
  - Vacuum solution of Einstein's equations (general relativity).
  - Unique solution with these properties (Birkhoff theorem).

Quantum corrected black hole

- Schwarzschild black hole:
  - Spherically symmetric spacetime.
  - Vacuum solution of Einstein's equations (general relativity).
  - Unique solution with these properties (Birkhoff theorem).
- κ-Poincaré modification of spacetime:
  - Interaction between particles and "quantum structure of spacetime".
  - Interaction depends on de Broglie wavelength (momentum).
  - $\rightsquigarrow$ Distinguished time direction (vector field).
  - $\Rightarrow$ κ-Minkowski spacetime has modified symmetry algebra.
  - Black hole spacetime: assume spherically symmetric vector field.
  - $\Rightarrow$ Vector field may only have time and radial components.
  - Modification depends on a parameter ℓ (Planck length).
  - Spacetime approaches Schwarzschild for $\ell \to 0$.

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^a\bar{x}_a\right) + \frac{1}{2}e^{\ell Z^a\bar{x}_a}(g^{ab} + Z^aZ^b)\bar{x}_a\bar{x}_b\,. \tag{8}$$

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^a\bar{x}_a\right) + \frac{1}{2}e^{\ell Z^a\bar{x}_a}(g^{ab} + Z^aZ^b)\bar{x}_a\bar{x}_b\,. \tag{8}$$

  - Spacetime metric g_{ab}.

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^a\bar{x}_a\right) + \frac{1}{2}e^{\ell Z^a\bar{x}_a}(g^{ab} + Z^aZ^b)\bar{x}_a\bar{x}_b\,. \tag{8}$$

  - Spacetime metric g_{ab}.
  - Unit timelike vector field Z^a satisfying $Z^aZ^bg_{ab} = -1$.

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^a\bar{x}_a\right) + \frac{1}{2}e^{\ell Z^a\bar{x}_a}(g^{ab} + Z^aZ^b)\bar{x}_a\bar{x}_b\,. \tag{8}$$

  - Spacetime metric g_{ab}.
  - Unit timelike vector field Z^a satisfying $Z^aZ^bg_{ab} = -1$.
  - Planck length ℓ.

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^a\bar{x}_a\right) + \frac{1}{2}e^{\ell Z^a\bar{x}_a}(g^{ab} + Z^aZ^b)\bar{x}_a\bar{x}_b\,. \tag{8}$$

  - Spacetime metric g_{ab}.
  - Unit timelike vector field Z^a satisfying $Z^aZ^bg_{ab} = -1$.
  - Planck length ℓ.

$\Rightarrow$ Static spherically symmetric case defined by functions a, b, c, d of r:

$$H = -\frac{2}{\ell^2}\sinh^2\left[\frac{\ell}{2}(-cE + dP)\right] + \frac{1}{2}e^{\ell(-cE+dP)}\left[(-a + c^2)E^2 - 2cdEP + (b + d^2)P^2 + \frac{L^2}{r^2}\right]. \tag{9}$$

κ-Poincaré correction of Schwarzschild spacetime

- General κ-Poincaré modification of metric dispersion relation:

$$H = -\frac{2}{\ell^2}\sinh^2\left(\frac{\ell}{2}Z^a\bar{x}_a\right) + \frac{1}{2}e^{\ell Z^a\bar{x}_a}(g^{ab} + Z^aZ^b)\bar{x}_a\bar{x}_b\,. \tag{8}$$

  - Spacetime metric g_{ab}.
  - Unit timelike vector field Z^a satisfying $Z^aZ^bg_{ab} = -1$.
  - Planck length ℓ.

⇒ Static spherically symmetric case defined by functions a, b, c, d of r:

$$H = -\frac{2}{\ell^2}\sinh^2\left[\frac{\ell}{2}(-cE + dP)\right] + \frac{1}{2}e^{\ell(-cE+dP)}\left[(-a + c^2)E^2 - 2cdEP + (b + d^2)P^2 + \frac{L^2}{r^2}\right]\,. \tag{9}$$

⇒ Minimal modification of Schwarzschild spacetime of mass M:

$$a^{-1} = b = c^{-2} = 1 - \frac{2M}{r}\,, \quad d = 0\,. \tag{10}$$

Example: swarm of orbiting particles

- Properties of particle ensemble:
  - Identical angular momentum $L > 0$ (motion has angular component).
  - Energy E such that particles are gravitationally bound.

Example: swarm of orbiting particles

- Properties of particle ensemble:
  - Identical angular momentum $L > 0$ (motion has angular component).
  - Energy E such that particles are gravitationally bound.

$\Rightarrow$ Orbit oscillates between two radii $R_{1,2}$.

Example: swarm of orbiting particles

- Properties of particle ensemble:
  - Identical angular momentum $L > 0$ (motion has angular component).
  - Energy E such that particles are gravitationally bound.

$\Rightarrow$ Orbit oscillates between two radii $R_{1,2}$.

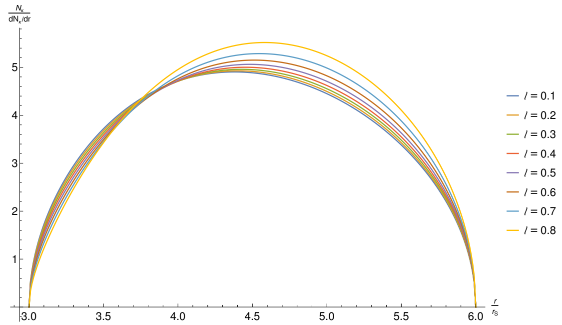
- Calculate number of trajectories through time slice σ with $R < r < R + \mathrm{d}r$.

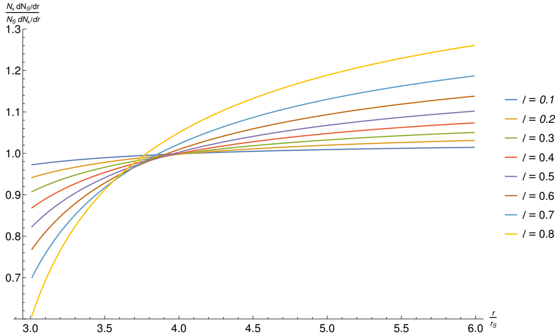
Example: swarm of orbiting particles

- Properties of particle ensemble:
  - Identical angular momentum $L > 0$ (motion has angular component).
  - Energy E such that particles are gravitationally bound.

$\Rightarrow$ Orbit oscillates between two radii $R_{1,2}$.

- Calculate number of trajectories through time slice σ with $R < r < R + \mathrm{d}r$.
- Plot (inverse of) relative particle density $N/(\mathrm{d}N/\mathrm{d}r)$:



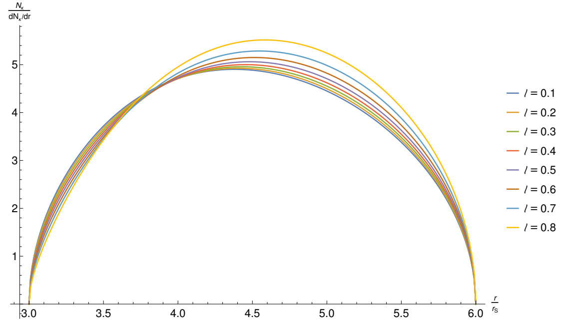


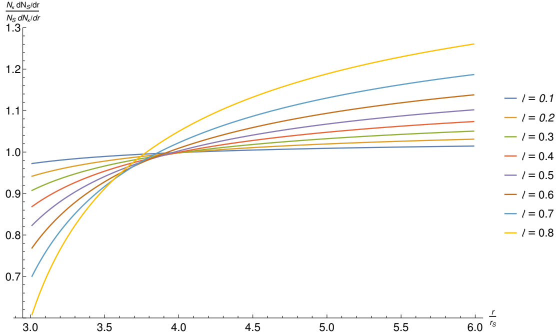
Example: swarm of orbiting particles

- Properties of particle ensemble:
  - Identical angular momentum $L > 0$ (motion has angular component).
  - Energy E such that particles are gravitationally bound.

$\Rightarrow$ Orbit oscillates between two radii $R_{1,2}$.

- Calculate number of trajectories through time slice σ with $R < r < R + \mathrm{d}r$.
- Plot (inverse of) relative particle density $N/(\mathrm{d}N/\mathrm{d}r)$:





$\Rightarrow$ κ-Poincaré modification shifts particles inward.

Example: radial free fall from infinity

- Properties of particle ensemble:
  - Identical angular momentum $L = 0$ (purely radial motion).
  - Energy E such that particles are marginally bound (drop from rest at $r = \infty$).

Example: radial free fall from infinity

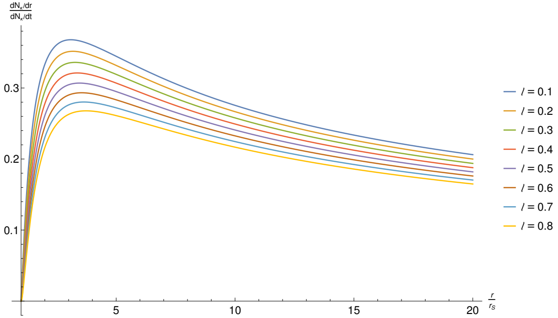
- Properties of particle ensemble:
  - Identical angular momentum $L = 0$ (purely radial motion).
  - Energy E such that particles are marginally bound (drop from rest at $r = \infty$).
- Assume constant flow rate through radial slice.

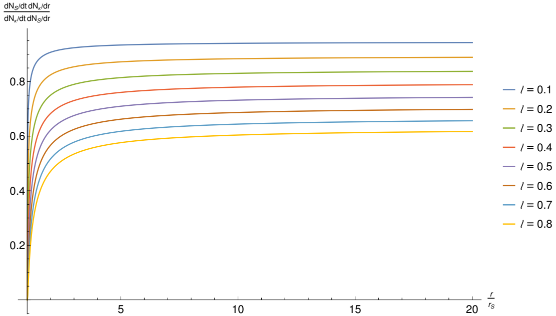
Example: radial free fall from infinity

- Properties of particle ensemble:
    - Identical angular momentum $L = 0$ (purely radial motion).
    - Energy E such that particles are marginally bound (drop from rest at $r = \infty$).
- Assume constant flow rate through radial slice.
- Calculate number of trajectories through time slice σ with $R < r < R + \mathrm{d}r$.

Example: radial free fall from infinity

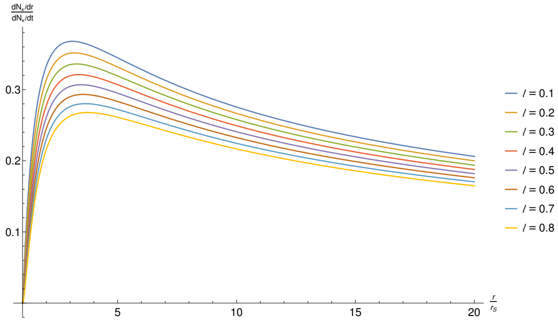
- Properties of particle ensemble:
  - Identical angular momentum $L = 0$ (purely radial motion).
  - Energy E such that particles are marginally bound (drop from rest at $r = \infty$).
- Assume constant flow rate through radial slice.
- Calculate number of trajectories through time slice σ with $R < r < R + \mathrm{d}r$.
- Plot particle density $\mathrm{d}N/\mathrm{d}r$ per flow rate $\mathrm{d}N/\mathrm{d}t$:

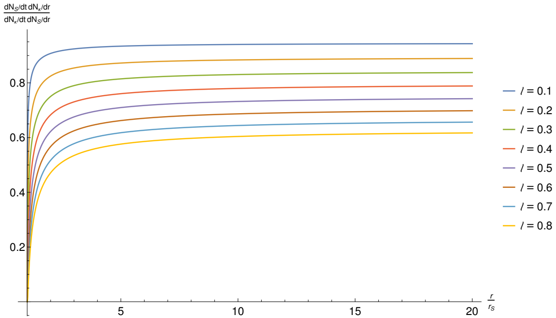




Example: radial free fall from infinity

- Properties of particle ensemble:
  - Identical angular momentum $L = 0$ (purely radial motion).
  - Energy E such that particles are marginally bound (drop from rest at $r = \infty$).
- Assume constant flow rate through radial slice.
- Calculate number of trajectories through time slice σ with $R < r < R + \mathrm{d}r$.
- Plot particle density $\mathrm{d}N/\mathrm{d}r$ per flow rate $\mathrm{d}N/\mathrm{d}t$:





$\Rightarrow$ κ-Poincaré modification decreases particle density.

Overview

Cosmological symmetry

- Introduce suitable coordinates on TM:

$$t, r, \theta, \varphi, y^t, y^r, y^\theta, y^\varphi \, .$$

Cosmological symmetry

- Introduce suitable coordinates on TM:

$$t, r, \theta, \varphi, y^t, y^r, y^\theta, y^\varphi \,.$$

- Most general Finsler function obeying cosmological symmetry:

$$F = F(t, y^t, w)\,, \quad w^2 = \frac{(y^r)^2}{1 - kr^2} + r^2 \left((y^\theta)^2 + \sin^2\theta (y^\varphi)^2\right)\,.$$

- Homogeneity of Finsler function $F(t, y^t, w) = y^t f(t, w/y^t)$.

Cosmological symmetry

- Introduce suitable coordinates on TM:

$$t, r, \theta, \varphi, y^t, y^r, y^\theta, y^\varphi .$$

- Most general Finsler function obeying cosmological symmetry:

$$F = F(t, y^t, w), \quad w^2 = \frac{(y^r)^2}{1 - kr^2} + r^2 \left((y^\theta)^2 + \sin^2\theta (y^\varphi)^2 \right) .$$

- Homogeneity of Finsler function $F(t, y^t, w) = y^t f(t, w/y^t)$.
- Introduce new coordinates: $\tilde{y} = y^t f(t, w/y^t)$, $s = w/y^t$.

$\Rightarrow$ Coordinates on observer space O with $\tilde{y} \equiv 1$.

$\Rightarrow$ Geometry function $f(t, s)$ on O.

Cosmological fluid dynamics

- Most general fluid obeying cosmological symmetry:

$$\phi = \phi(t, s)\,.$$

Cosmological fluid dynamics

- Most general fluid obeying cosmological symmetry:

$$\phi = \phi(t, s) \,.$$

- Collisionless fluid satisfies Liouville equation [MH '15]:

$$0 = \mathcal{L}_{\mathbf{r}}\phi = \frac{1}{f}\left(\partial_t\phi - \frac{\partial_t\partial_s f}{\partial_s^2 f}\partial_s\phi\right) \,.$$

Cosmological fluid dynamics

- Most general fluid obeying cosmological symmetry:

$$\phi = \phi(t, s)\,.$$

- Collisionless fluid satisfies Liouville equation [MH '15]:

$$0 = \mathcal{L}_{\mathbf{r}}\phi = \frac{1}{f}\left(\partial_t\phi - \frac{\partial_t\partial_s f}{\partial_s^2 f}\partial_s\phi\right)\,.$$

- Example: collisionless dust fluid $\phi(x, y) \sim \rho(x)\delta_{S_x}(y, u(x))$:

$$u(t) = \frac{1}{f(t,0)}\partial_t\,, \quad \partial_t\left(\rho(t)\sqrt{g^F(t,0)}\right) = 0\,.$$

Ongoing project: Cosmological solutions

- Slowly moving kinetic gas: $\phi(t, s)$ concentrated around $s = 0$.

Ongoing project: Cosmological solutions

- Slowly moving kinetic gas: $\phi(t, s)$ concentrated around $s = 0$.
- High velocity limit: $\lim_{s \to 1} \phi(t, s) = 0$.

Ongoing project: Cosmological solutions

- Slowly moving kinetic gas: $\phi(t, s)$ concentrated around $s = 0$.
- High velocity limit: $\lim_{s \to 1} \phi(t, s) = 0$.
- Expanding universe leads to cooling.

Ongoing project: Cosmological solutions

- Slowly moving kinetic gas: $\phi(t, s)$ concentrated around $s = 0$.
- High velocity limit: $\lim_{s \to 1} \phi(t, s) = 0$.
- Expanding universe leads to cooling.

⇒ Possible to decompose gravity equation with source:

$$\phi(t, s) = \sum_{k=0}^{\infty} \frac{1}{k!} \phi_k(t) s^k \,.$$

Ongoing project: Cosmological solutions

- Slowly moving kinetic gas: $\phi(t, s)$ concentrated around $s = 0$.
- High velocity limit: $\lim_{s \to 1} \phi(t, s) = 0$.
- Expanding universe leads to cooling.

⇒ Possible to decompose gravity equation with source:

$$\phi(t, s) = \sum_{k=0}^{\infty} \frac{1}{k!} \phi_k(t) s^k \,.$$

⇒ Study Finsler function given by Taylor series:

$$f(t, s) = \sum_{k=0}^{\infty} \frac{1}{k!} f_k(t) s^k \,.$$

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.
- Embedded hypersurface: graph of function $u : Q \to \mathbb{R}$.

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.
- Embedded hypersurface: graph of function $u : Q \to \mathbb{R}$.
- Q equipped with positive definite Finsler function $W : TQ \to \mathbb{R}^+$.

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.
- Embedded hypersurface: graph of function $u : Q \to \mathbb{R}$.
- Q equipped with positive definite Finsler function $W : TQ \to \mathbb{R}^+$.
- Ansatz for Finsler function on M as $F^2 = -\bar{t}^2 + f^2(t)W(v)$.

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.
- Embedded hypersurface: graph of function $u : Q \to \mathbb{R}$.
- Q equipped with positive definite Finsler function $W : TQ \to \mathbb{R}^+$.
- Ansatz for Finsler function on M as $F^2 = -\bar{t}^2 + f^2(t)W(v)$.

$\Rightarrow$ Normal vector $n \in TM$ determined by Legendre transformation.

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.
- Embedded hypersurface: graph of function $u : Q \to \mathbb{R}$.
- Q equipped with positive definite Finsler function $W : TQ \to \mathbb{R}^+$.
- Ansatz for Finsler function on M as $F^2 = -\bar{t}^2 + f^2(t)W(v)$.

$\Rightarrow$ Normal vector $n \in TM$ determined by Legendre transformation.

$\Rightarrow$ $g^F(n)$ determines Riemannian metric on Q.

Upcoming project: Cauchy surfaces

- Open question: Cauchy surfaces for Finsler gravity dynamics?
- Idea: consider globally hyperbolic spacetime $M = Q \times \mathbb{R}$.
- Embedded hypersurface: graph of function $u : Q \to \mathbb{R}$.
- Q equipped with positive definite Finsler function $W : TQ \to \mathbb{R}^+$.
- Ansatz for Finsler function on M as $F^2 = -\bar{t}^2 + f^2(t)W(v)$.

$\Rightarrow$ Normal vector $n \in TM$ determined by Legendre transformation.

$\Rightarrow$ $g^F(n)$ determines Riemannian metric on Q.

$\rightsquigarrow$ Determine mean curvature flow and Cauchy hypersurface property.

Overview

Conclusion

- Summary:
  - ↯ Fundamental theory of quantum gravity is unknown.
  - ⇒ Consider effective quantum gravity models instead.
  - ○ Effective model is small correction to general relativity.
  - ⇒ Study observable effects of possible quantum corrections.
  - ○ κ-Poincaré modification changes matter density near black hole.

Conclusion

- Summary:
  - ↯ Fundamental theory of quantum gravity is unknown.
  - $\Rightarrow$ Consider effective quantum gravity models instead.
  - ○ Effective model is small correction to general relativity.
  - $\Rightarrow$ Study observable effects of possible quantum corrections.
  - ○ κ-Poincaré modification changes matter density near black hole.
- Outlook:
  - ○ Consider more general quantum corrections.
  - ○ Consider spinning black holes.
  - ○ Consider more general gases or matter distributions with less symmetry:
    - Accretion disks and jets $\rightsquigarrow$ blazars.
    - Tidal disruption events.
    - Stellar wake of passing black hole and dynamical friction.
  - ○ Derive observable properties of black holes, quasars, AGN...

Conclusion

- Summary:
  - ↯ Fundamental theory of quantum gravity is unknown.
  - ⇒ Consider effective quantum gravity models instead.
  - ○ Effective model is small correction to general relativity.
  - ⇒ Study observable effects of possible quantum corrections.
  - ○ κ-Poincaré modification changes matter density near black hole.
- Outlook:
  - ○ Consider more general quantum corrections.
  - ○ Consider spinning black holes.
  - ○ Consider more general gases or matter distributions with less symmetry:
    - Accretion disks and jets ⇝ blazars.
    - Tidal disruption events.
    - Stellar wake of passing black hole and dynamical friction.
  - ○ Derive observable properties of black holes, quasars, AGN...
- MH, "Kinetic gases in static spherically symmetric modified dispersion relations," Class. Quant. Grav. **41** (2024) no.1, 015025 [arXiv:2310.01487 [gr-qc]].