Repulsive gravity model for dark energy

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Foundational Aspects of Cosmology 18 February 2011

Outline

Motivation

- 2 Multimetric gravity
- 3 Multimetric cosmology
 - Simulation of structure formation
- 5 Solar system consistency
- 6 Gravitational waves

7 Conclusion

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22.8% dark matter.

Galaxy rotation curves.

[de Blok, Bosma '02]

Anomalous light deflection.

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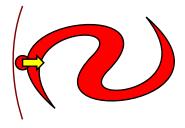
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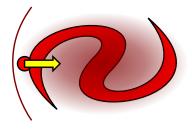
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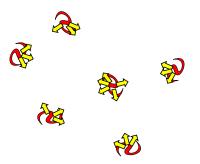
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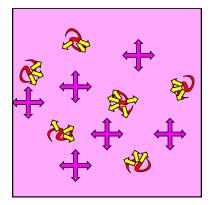
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\Rightarrow Problem: What are dark matter and dark energy?

Explanations for the dark universe

• Particle physics:

- Dark matter: [Bertone, Hooper, Silk '05]
 - Weakly interacting massive particles (WIMPs). [Ellis et al. '84]
 - Axions. [Preskill, Wise, Wilczek '83]
 - Massive compact halo objects (MACHOs). [Paczynski '86]
- Dark energy: [Copeland, Sami, Tsujikawa '06]
 - Quintessence. [Peebles, Ratra '88]
 - K-essense. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
 - Chaplygin gas. [Kamenshchik, Moschella, Pasquier '01]
- Gravity:
 - Modified Newtonian dynamics (MOND). [Milgrom '83]
 - Tensor-vector-scalar theories. [Bekenstein '04]
 - Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
 - Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
 - Non-symmetric gravity. [Moffat '95]
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 - Non-symmetric gravity. [Moffat '95]
 - Area metric gravity. [Punzi, Schuller, Wohlfarth '07]
 - New idea: repulsive gravity \Leftrightarrow negative mass!

Three types of mass! [Bondi '57]

- Active gravitational mass m_a source of gravity: $\phi = -G_N \frac{m_a}{r}$.
- Passive gravitational mass m_p reaction on gravity: $\vec{F} = -m_p \vec{\nabla} \phi$.
- Inertial mass m_i relates force to acceleration: $\vec{F} = m_i \vec{a}$.

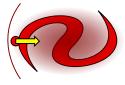
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- $m_a \sim m_p \sim m_i$ experimentally confirmed.
- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.

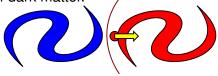
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- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.
- Observations exist for visible matter only.

- Idea for dark universe: standard model with $m_a = m_p = -m_i < 0$.
- Both copies couple only through gravity \Rightarrow "dark".
- \Rightarrow Preserves momentum conservation.
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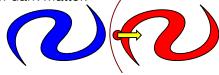
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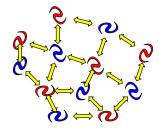
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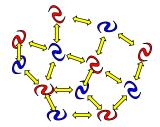
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- Explanation of dark energy.
- $\Rightarrow \text{ Advantage: Dark copy } \Psi^- \text{ of } \\ \text{ well-known standard model } \Psi^+:$
 - No new parameters.
 - No unknown masses.
 - No unknown couplings.



- Positive and negative test masses follow different trajectories.
- Two types of test mass trajectories \Rightarrow two types of observers.
- Observer trajectories are autoparallels of two connections ∇[±].
- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors g[±]_{ab}.

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- Frames are orthonormalized using two metric tensors g[±]_{ab}.
- No-go theorem forbids bimetric repulsive gravity. [MH, M. Wohlfarth '09]
- Solution: $N \ge 3$ metrics g_{ab}^l and standard model copies Ψ^l .

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- 1. $N \ge 3$ copies of standard model matter Ψ^{I} , I = 1, ..., N.
- 2. *N* metric tensors g_{ab}^{l} .
- 3. Each standard model copy Ψ^{\prime} couples only to its metric g^{\prime} .

$$\Rightarrow \quad S_M[g',\Psi'] = \int d^4x \sqrt{g'} \mathcal{L}_M[g',\Psi'].$$

4. Different sectors couple only gravitationally.

$$\Rightarrow \quad S = S_G[g^1, \dots, g^N] + \sum_{l=1}^N S_M[g^l, \Psi^l].$$

- 5. Field equations contain at most second derivatives of the metrics.
- 6. Symmetric with respect to permutations of the sectors (g^{l}, Ψ^{l}) .

Action and equations of motion

Gravitational action:

$$S_G[g^1, \dots, g^N] = rac{1}{2} \int d^4 x \sqrt{g_0} \left[\sum_{I,J=1}^N (x + y \delta^{IJ}) g^{Iij} R^J_{ij} + F(S^{IJ})
ight]$$

- Symmetric volume form $g_0 = (g^1 g^2 \dots g^N)^{1/N}$.
- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^{I} \Gamma^{J}$.

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- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^{I} \Gamma^{J}$.
- \Rightarrow Equations of motion:

$$T'_{ab} = \sqrt{\frac{g_0}{g'}} \left[-\frac{1}{2N} g'_{ab} \sum_{J,K=1}^{N} (x + y\delta^{JK}) g^{Jij} R^{K}_{ij} + \sum_{J=1}^{N} (x + y\delta^{JJ}) R^{J}_{ab} \right]$$

+ terms linear in $\nabla^{I} S^{JK}$

- + terms quadratic in S^{IJ} .
- ⇒ Repulsive Newtonian limit for $N \ge 3$.

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Standard cosmology: Robertson–Walker metrics

$$g^{I}=-n_{I}^{2}(t)dt\otimes dt+a_{I}^{2}(t)\gamma_{lphaeta}dx^{lpha}\otimes dx^{eta}$$

- Lapse functions *n*_l.
- Scale factors *a*_l.
- Spatial metric $\gamma_{\alpha\beta}$ of constant curvature $k \in \{-1, 0, 1\}$ and Riemann tensor $R(\gamma)_{\alpha\beta\gamma\delta} = 2k\gamma_{\alpha[\gamma}\gamma_{\delta]\beta}$.
- Perfect fluid matter:

$$T^{Iab} = (\rho_I + p_I)u^{Ia}u^{Ib} + p_I g^{Iab}.$$

• Normalization: $g_{ab}^{l}u^{la}u^{lb} = -1$.

Simple cosmological model

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- \Rightarrow Single effective energy-momentum tensor $T_{ab}^{l} = T_{ab}$.
- \Rightarrow Single effective metric $g_{ab}^{\prime} = g_{ab}$.
- \Rightarrow Common scale factors a' = a and lapse functions n' = n.
- \Rightarrow Rescale cosmological time to set $n \equiv 1$.
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- \Rightarrow Ricci tensors $R_{ab}^{l} = R_{ab}$ become equal.
- \Rightarrow Connection differences $S^{IJi}_{jk} = 0$ vanish.
- \Rightarrow Equations of motion simplify for repulsive Newtonian limit:

$$(2-N)T_{ab}=R_{ab}-\frac{1}{2}Rg_{ab}.$$

 \Rightarrow Negative effective gravitational constant for early / late universe.

Insert Robertson–Walker metric into equations of motion:

$$\rho = \frac{3}{2 - N} \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right),$$
$$\rho = -\frac{1}{2 - N} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right).$$

- ⇒ Positive matter density ρ > 0 requires k = -1 and $\dot{a}^2 < 1$.
- \Rightarrow No solutions for k = 0 or k = 1.

Accelerating expansion

• Acceleration equation:

$$\frac{\ddot{a}}{a}=\frac{N-2}{6}\left(\rho+3p\right).$$

- Factor N 2 > 0 for multimetric gravity.
- Strong energy condition

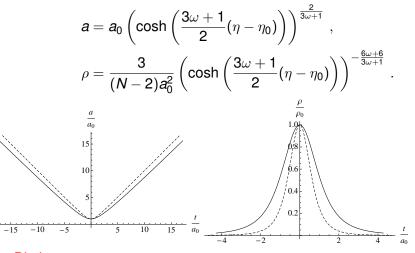
$$\left(T_{ab}-\frac{1}{2}Tg_{ab}\right)t^{a}t^{b}\geq0$$

for all timelike vector fields t^a implies $\rho + 3p \ge 0$.

 \Rightarrow Acceleration must be positive.

Explicit solution

Equation of state: p = ωρ; dust: ω = 0, radiation: ω = 1/3.
General solution using conformal time η defined by dt = a dη:



 \Rightarrow Big bounce at $\eta=\eta_{0}$. [MH, M. Wohlfarth '10]

Manuel Hohmann (Uni Hamburg)

Repulsive gravity & cosmology

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Ingredients

1. Metrics
$$g_{ab}^{\prime}=g_{ab}^{0}+h_{ab}^{\prime}$$
 with

$$g^0 = - dt \otimes dt + a^2(t) \gamma_{lphaeta} dx^lpha \otimes dx^eta$$

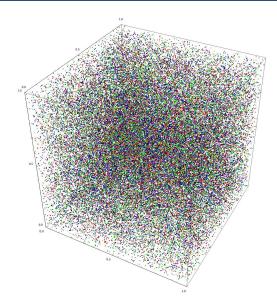
and a(t) determined by cosmology.

- 2. Scale for structure formation \ll curvature radius of the universe:
 - Cubic volume $0 \le x^{\alpha} \le \ell$.
 - Approximate $\gamma_{\alpha\beta}$ by $\delta_{\alpha\beta}$.
 - Periodic boundary conditions.
- 3. Matter content: *n* point masses *M* for each sector.
 - Model for dust matter: p = 0.
 - Matter density:

$$\rho = \frac{Mn}{(a\ell)^3} \, .$$

- 4. Large mean distance $a\ell/\sqrt[3]{Nn} \gg 2GM$.
- 5. Small velocities $|v_{li}^{\alpha}| = |a\dot{x}_{li}^{\alpha}| \ll 1$.

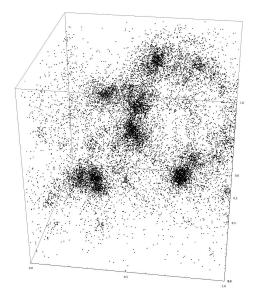
Evolution of structures for all matter types



- *N* = 4.
- *n* = 16384.
- 7.5 days CPU time.

Final state for one matter type

- Galactic clusters.
- Empty voids.



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• Consider only repulsive gravity between different sectors.

- \Rightarrow Different matter types should separate.
- ⇒ Energy-momentum tensor contains only visible matter:

$$\Rightarrow T_{ab}^+ = T_{ab}^1 \neq 0. \Rightarrow T_{ab}^- = T_{ab}^2 = \ldots = T_{ab}^N = 0.$$

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- Permutation symmetry between sectors.
 - \Rightarrow Visible matter has equal effects on all dark sectors.
 - \Rightarrow Metric:

$$\begin{array}{l} \Rightarrow \hspace{0.1cm} g_{ab}^{+} = g_{ab}^{1}.\\ \Rightarrow \hspace{0.1cm} g_{ab}^{-} = g_{ab}^{2} = \ldots = g_{ab}^{N}. \end{array}$$

Parametrized post-Newtonian formalism

- Characterize single-metric gravity theories by 10 parameters. [Thorne, Will '71; Will '93]
- 2 parameters can be obtained from linearized field equations.
- Values constrained by experiment, e.g., $\gamma = 1 \pm 2.3 \cdot 10^{-5}$.

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- 2 parameters can be obtained from linearized field equations.
- Values constrained by experiment, e.g., $\gamma = 1 \pm 2.3 \cdot 10^{-5}$.
- Extension of PPN formalism for multimetric gravity theories. [MH, M. Wohlfarth '10]
- Extended set of 26 PPN parameters.
- 8 parameters can be obtained from linearized field equations.
- 10 parameters correspond to single-metric parameters ⇒ experimentally measured.

PPN consistent theory

Consider gravitational action of the form:

$$S_G = \frac{1}{2} \int d^4x \sqrt{g_0} \sum_{l=1}^N g^{l\,ij} \left[z \tilde{S}^I_k \tilde{S}^{l\,k}_{\ ij} + u \tilde{S}^I_i \tilde{S}^I_j + \sum_{J=1}^N (x + y \delta^{JJ}) R^J_{ij} \right]$$

• Parameters in the action:

$$x = \frac{1}{8 - 4N}$$
, $y = \frac{4 - N}{8 - 4N}$, $z = -\frac{4 - N}{8 - 4N}$, $u = -\frac{12 - 3N}{8 - 4N}$

 \Rightarrow PPN parameters from linearized formalism:

- $\alpha^+ = 1$, $\theta^+ = 0$: standard PPN gauge choice.
- $\gamma^+ = 1$, $\sigma^+_+ = -2$: experimental consistency.
- $\alpha^{-} = -1$: repulsive Newtonian limit.
- $\gamma^{-} = -1$, $\theta^{-} = 0$, $\sigma^{-}_{+} = 2$: additional "dark" PPN parameters.

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Propagation velocity

• Write linearized vacuum field equations as:

$$\sum_{J=1}^N \mathcal{D}^{IJ}_{\ ab}{}^{cd}h^J_{cd} = 0\,.$$

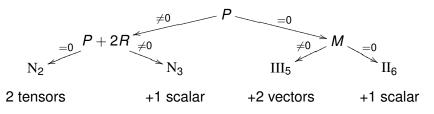
- Differential operator \mathcal{D} .
- Consider wavelike solution:

$$h_{ab}^{\prime}=h_{ab}^{\prime\,0}e^{ik_ax^a}$$

- Multiplication operator $\hat{\mathcal{D}}$.
- Non-vanishing solution require zero eigenvalue of $\hat{\mathcal{D}}$.
- det $\hat{\mathcal{D}} \propto (k^a k_a)^{10}$.
- \Rightarrow Solutions exist only for $k^a k_a = 0$.
- \Rightarrow Gravitational waves are null.

Polarization and E(2) class

- Up to 6 polarizations in general metric theories.
- Theories classified by representations of E(2).
- E(2) class of multimetric gravity depends on 3 parameters:



- PPN consistent theory shown before of class N₂.
- \Rightarrow Same class as general relativity.

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- Idea here: Dark universe might be explained by repulsive gravity.
- \Rightarrow Repulsive gravity requires an extension of general relativity.
- \Rightarrow No-go theorem: bimetric repulsive gravity is not possible.
- \Rightarrow Multimetric repulsive gravity with $N \ge 3$ by explicit construction.
- $\Rightarrow\,$ Cosmology features late-time acceleration and big bounce.
- \Rightarrow Structure formation features clusters and voids.
- $\Rightarrow\,$ Repulsive gravity is consistent with solar system experiments.
- \Rightarrow Gravitational waves are null.

Outlook

- Remaining PPN parameters should be determined from full multimetric PPN formalism.
- Restrict multimetric gravity theories by additional PPN bounds.
- Establish further construction principles, e.g., continuous symmetry between sectors.
- Examine initial-value problem.
- Determine further exact solutions (single point mass...).
- Advanced simulation of structure formation including thermodynamics using GADGET-2 (Millenium Simulation).
- Search for repulsive gravity sources in the galactic voids through gravitational lensing.
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