Multimetric repulsive gravity

Manuel Hohmann and Mattias N. R. Wohlfarth

Emmy Noether-Programm
Deutsche Forschungsgemeinschaft DFG

Universität Hamburg
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Motivation

- Only 5% of the universe are visible.
- 95% are completely unknown.
- Idea: Add negative mass standard model.
- Both copies appear mutually dark.
- Well-known masses and couplings.

Repulsive Einstein gravity

- Positive and negative test masses.
- Two types of observer trajectories.
- Autoparallels of two connections $\nabla^{\pm}$.
- Attach parallely transported frames.
- Orthonormalize with two metrics $g^{\pm}_{ab}$.
- Bimetric repulsive gravity not possible [1].
- Solution: *Multi*metric gravity.

Multimetric gravity

- $N \geq 3$ metric tensors g^I_{ab}.
- N standard model copies Ψ^I:

$$\Rightarrow S_M[g^I, \Psi^I] = \int d^4x \sqrt{g^I} \mathcal{L}_M[g^I, \Psi^I] .$$

- SM copies couple only to their metrics:

$$\Rightarrow \quad S = S_G[g^1, \ldots, g^N] + \sum_{I=1}^{N} S_M[g^I, \Psi^I] .$$

- Repulsive Newtonian limit:

$$\triangle \begin{pmatrix} \Phi^1 \\ \Phi^2 \\ \vdots \\ \Phi^N \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & & -1 \\ \vdots & & \ddots & \\ -1 & -1 & & 1 \end{pmatrix} \begin{pmatrix} \rho^1 \\ \rho^2 \\ \vdots \\ \rho^N \end{pmatrix} .$$

Post-Newtonian limit

- Parametrized post-Newtonian formalism.
- Extension to multimetric gravity.
- 26 PPN parameters.
- 8 parameters from linearized equations.
- Example for gravitational action [3]:

$$S_G = \frac{1}{16 - 8N} \int d^4x \sqrt{g_0} \sum_{I=1}^{N} g^{I\,ij} \Big[z \tilde{S}^I{}_k \tilde{S}^{I\,k}{}_{ij} + u \tilde{S}^I{}_i \tilde{S}^I{}_j + \sum_{J=1}^{N} (x + y\delta^{IJ}) R^J_{ij} \Big] .$$

- PPN consistency requires:

$$x = 1 , \quad -y = z = N - 4 , \quad u = 3N - 12 .$$

- PPN parameters:

$$\alpha^+ = 1 , \quad \gamma^+ = 1 , \quad \theta^+ = 0 , \quad \sigma^+_+ = -2 ,$$
$$\alpha^- = -1 , \quad \gamma^- = -1 , \quad \theta^- = 0 , \quad \sigma^-_+ = 2 .$$

References

[1] M. Hohmann and M. N. R. Wohlfarth, Phys. Rev. D **80** (2009) 104011 [arXiv:0908.3384 [gr-qc]].

[2] M. Hohmann and M. N. R. Wohlfarth, Phys. Rev. D **81** (2010) 104006 [arXiv:1003.1379 [gr-qc]].

[3] M. Hohmann and M. N. R. Wohlfarth, Phys. Rev. D **82** (2010) 084028 [arXiv:1007.4945 [gr-qc]].

Cosmology

- Standard cosmology defined by Robertson–Walker metrics and perfect fluid matter .
- Copernican principle: common evolution for all matter sectors.
- Effective equations of motion simplify: $(2-N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}$.
- Cosmological equations of motion ⇒ positive energy density ρ requires $k = -1$:

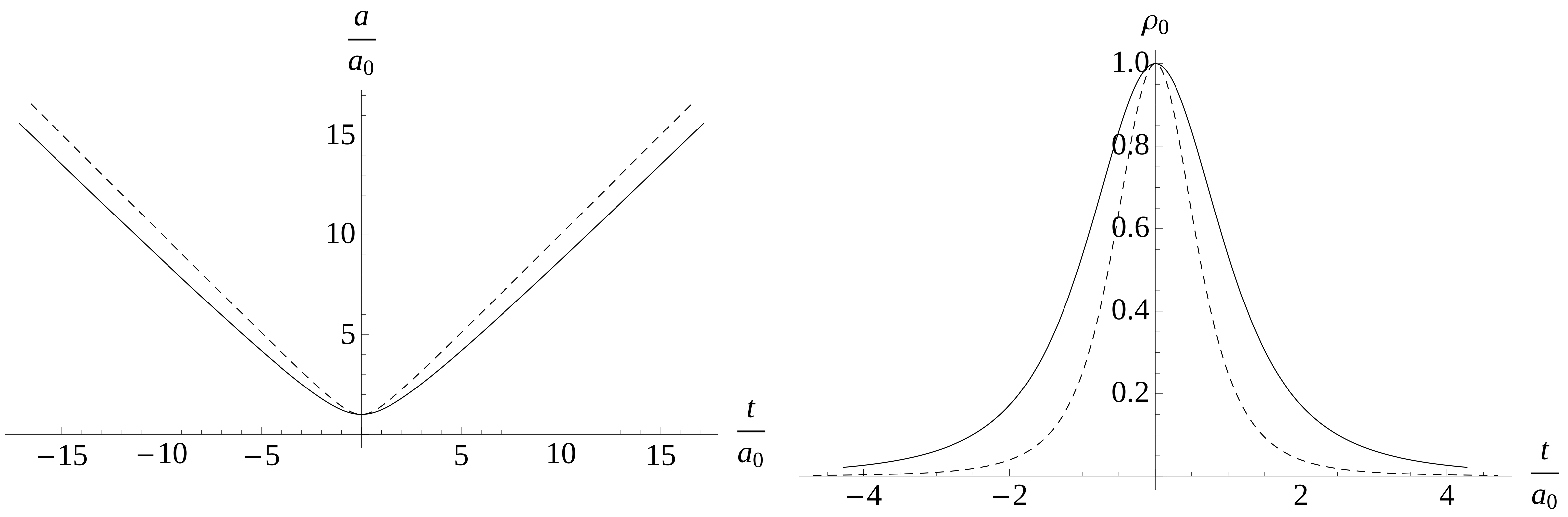
$$\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) , \quad p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) .$$

- Acceleration equation ⇒ acceleration must be positive:

$$\frac{\ddot{a}}{a} = \frac{N-2}{6}(\rho + 3p) > 0 .$$

- Equation of state: $p = \omega\rho$ with $\omega = 0$ for dust and $\omega = \frac{1}{3}$ for radiation.
- General solution using conformal time η defined by $dt = a\,d\eta$ [2]:

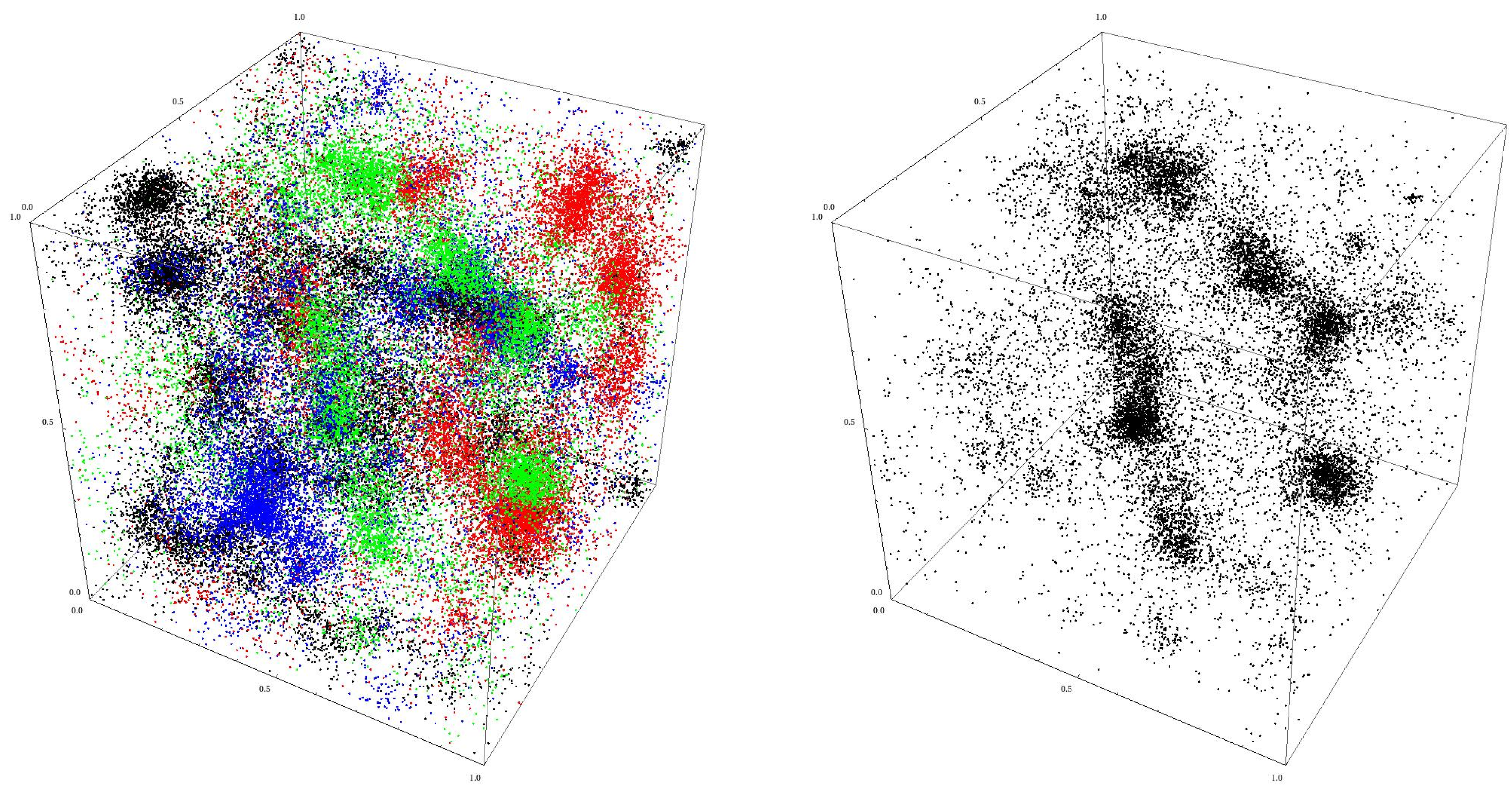
$$a = a_0 \left(\cosh\left(\frac{3\omega+1}{2}(\eta - \eta_0)\right)\right)^{\frac{2}{3\omega+1}} , \quad \rho = \frac{3}{(N-2)a_0^2}\left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{-\frac{6\omega+6}{3\omega+1}} .$$



Cosmological solution for $a(t)$ and $\rho(t)$ for dust (solid) and radiation (dashed).

Simulation of structure formation

- Based on cosmological background solution and repulsive Newtonian limit.
- Use n point masses of equal mass M for each matter type as model for dust matter.
- Treat gravitational interaction in Newtonian limit: large distances, small velocities.
- Numerical simulation with $N = 4$ and $n = 16384$, 7.5 days CPU time.



Final state of the simulation for all matter types (left) / only visible matter displayed (right).

Gravitational waves

- Perturbation around flat background metric: $g^I_{ab} = \eta_{ab} + h^I_{ab}$.
- Gravitational waves propagate at the speed of light.
- Up to 6 polarizations, determined on 3 parameters P, R, M: