Gravitational waves in multimetric gravity

An analysis of linearized multimetric gravity theories

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DER FORSCHUNG | DER LEHRE | DER BILDUNG

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Motivation

- ΛCDM model: 95% of the universe are dark matter / dark energy.
- Constituents of dark universe are unknown.
- Idea: DM / DE effects from additional *dark* standard model copies.
- Only interaction between standard model copies: repulsive gravity.
- Universe contains equal amounts of matter from all copies.

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- ΛCDM model: 95% of the universe are dark matter / dark energy.
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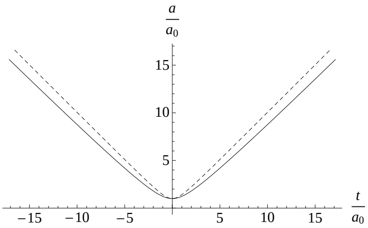
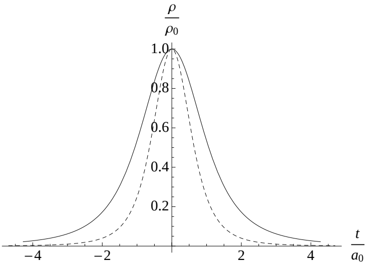
- Idea: DM / DE effects from additional *dark* standard model copies.
- Only interaction between standard model copies: repulsive gravity.
- Universe contains equal amounts of matter from all copies.
- Dark galaxies "push" visible matter & light towards visible galaxies.
  $\Rightarrow$ Explanation of dark matter!
- Mutual repulsion between galaxies drives accelerating expansion.
  $\Rightarrow$ Explanation of dark energy! [MH, M. Wohlfarth '10]





Construction principles

- $N \geq 2$ standard model copies Ψ^I governed by metrics g^I.
- Each standard model copy Ψ^I couples only to its own metric g^I:

$$\Rightarrow \quad S_M[g^I, \Psi^I] = \int d^4x \sqrt{g^I} \mathcal{L}_M[g^I, \Psi^I] \,.$$

- Different sectors couple only gravitationally:

$$\Rightarrow \quad S = S_G[g^1, \ldots, g^N] + \sum_{I=1}^{N} S_M[g^I, \Psi^I] \,.$$

- Field equations obtained from variation with respect to g^I:

$$K^I_{ab} = 8\pi G_N T^I_{ab}$$

- Curvature tensor K^I_{ab} of second derivative order.
- Permutation symmetry of the sectors (g^I, Ψ^I).
- Vacuum solution given by flat metrics $g^I = \eta$.

Linearized multimetric gravity

- Perturbation ansatz: $g^I = \eta + h^I$.
- Most general linearized geometry tensor:

$$\underline{K}_{ab} = \underline{\underline{P}} \cdot \partial^p \partial_{(a} \underline{h}_{b)p} + \underline{\underline{Q}} \cdot \Box \underline{h}_{ab} + \underline{\underline{R}} \cdot \partial_a \partial_b \underline{h} + \underline{\underline{M}} \cdot \partial^p \partial^q \underline{h}_{pq} \eta_{ab} + \underline{\underline{N}} \cdot \Box \underline{h} \eta_{ab}$$

- Parameter matrices $\underline{\underline{P}}, \underline{\underline{Q}}, \underline{\underline{R}}, \underline{\underline{M}}, \underline{\underline{N}}$.

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- Parameter matrices $\underline{\underline{P}}, \underline{\underline{Q}}, \underline{\underline{R}}, \underline{\underline{M}}, \underline{\underline{N}}$.
- Permutation symmetry of the field equations:

$$P^{IJ} = (P^+ - P^-)\delta^{IJ} + P^- \, , \ldots$$

- Simultaneously diagonalize parameter matrices.

$\Rightarrow$ Field equations decouple:

$$\mathfrak{K}^1_{ab} = P_1 \partial^p \partial_{(a} \mathfrak{h}^1_{b)p} + Q_1 \Box \mathfrak{h}^1_{ab} + R_1 \partial_a \partial_b \mathfrak{h}^1 + M_1 \partial^p \partial^q \mathfrak{h}^1_{pq} \eta_{ab} + N_1 \Box \mathfrak{h}^1 \eta_{ab}$$
$$\mathfrak{K}^i_{ab} = P_0 \partial^p \partial_{(a} \mathfrak{h}^i_{b)p} + Q_0 \Box \mathfrak{h}^i_{ab} + R_0 \partial_a \partial_b \mathfrak{h}^i + M_0 \partial^p \partial^q \mathfrak{h}^i_{pq} \eta_{ab} + N_0 \Box \mathfrak{h}^i \eta_{ab}$$

$\Rightarrow$ 10 parameters $P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0$.

Gauge invariance

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- Gravitational action:
  - Bianchi identity for $\mathfrak{h}^1$.
  - Geometric identity satisfied by *all* perturbations $\mathfrak{h}^1$.

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- Matter action:
  - Energy-momentum conservation: $\nabla^I_a T^{I\,ab} = 0$.
  - Field equations imply Bianchi identities for $\mathfrak{h}^i$.
  - Satisfied by all *solutions* $\mathfrak{h}^i$ of the field equations.
  - $\Rightarrow$ Not a geometric identity, no restriction on parameter matrices.

Propagation velocity

- Consider only physical degrees of freedom.
- Algebraic $3 + 1$-split and differential decomposition of $\mathfrak{h}^I_{ab}$:
  - $4N$ scalars $\mathfrak{I}^I_1, \mathfrak{I}^I_2, \mathfrak{I}^I_3, \mathfrak{I}^I_4$
  - $2N$ divergence-free vectors $\mathfrak{I}^I_\alpha, \mathfrak{I}'^I_\alpha$
  - N trace-free, divergence-free tensors $\mathfrak{I}^I_{\alpha\beta}$
- Gauge invariance:
  - $\mathfrak{I}^1_3, \mathfrak{I}^1_4, \mathfrak{I}'^1_\alpha$: pure gauge quantities
  - Remaining potentials $\mathfrak{I}$: gauge invariants
  - Curvature tensor $\mathfrak{K}^I_{ab}$ depends on gauge invariants only!

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- Consider wave-like solution:

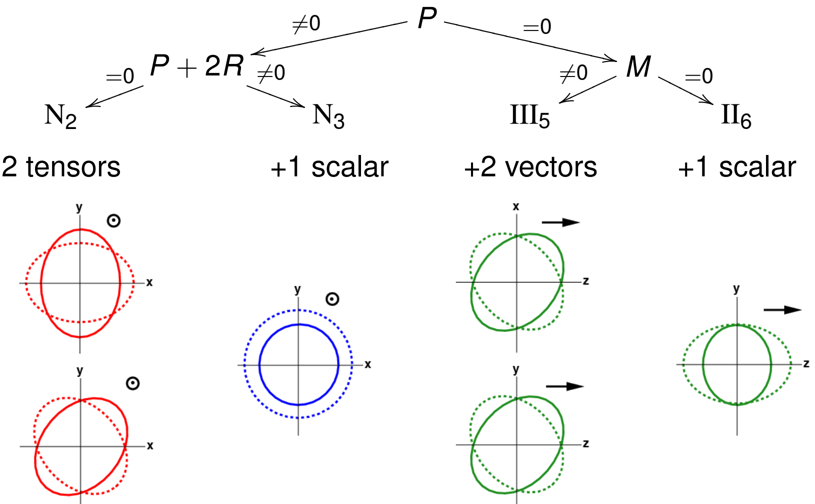
$$\mathfrak{I}(x) = \hat{\mathfrak{I}} e^{ik_a x^a}$$

$\Rightarrow$ Gravitational vacuum field equations solved only for $k_a k^a = 0$.

$\Rightarrow$ Gravitational waves propagate at the speed of light!

Polarizations and E(2) class

- Express Riemann tensor & $\mathfrak{h}^I$ in Newman-Penrose frame.
- Polarizations classified by reps. of E(2). [Eardley, Lee, Lightman *et al.* '73]
- E(2) class depends on P_1, R_1, M_1 for $\mathfrak{h}^1$ or P_0, R_0, M_0 for $\mathfrak{h}^i$.



A multimetric example

Gravitational action with parameters x, y, u, v, w, r, s [MH ⇝ PRD 03/12]:

$$S_G = \frac{1}{16\pi}\int d^4x\sqrt{g_0}\left[x\sum_{I,J=1}^{N} g^{I\,ij}R^{J}{}_{ij} + \sum_{I=1}^{N} g^{I\,ij}\left(yR^{I}{}_{ij} + u\tilde{S}^{I}{}_{i}\tilde{S}^{I}{}_{j}\right.\right.$$

$$\left.\left. + v\tilde{S}^{I}{}_{k}\tilde{S}^{I\,k}{}_{ij} + w\tilde{S}^{I\,k}{}_{im}\tilde{S}^{I\,m}{}_{jk} + g^{I\,kl}g^{I}{}_{mn}\left(r\tilde{S}^{I\,m}{}_{ik}\tilde{S}^{I\,n}{}_{jl} + s\tilde{S}^{I\,m}{}_{ij}\tilde{S}^{I\,n}{}_{kl}\right)\right)\right].$$

Connection difference tensors:

$$S^{IJ\,i}{}_{jk} = \Gamma^{I\,i}{}_{jk} - \Gamma^{J\,i}{}_{jk}\,, \qquad S^{IJ}{}_{j} = S^{IJ\,k}{}_{jk}\,,$$

$$\tilde{S}^{J\,i}{}_{jk} = \frac{1}{N}\sum_{I=1}^{N} S^{IJ\,i}{}_{jk}\,, \qquad \tilde{S}^{J}{}_{j} = \tilde{S}^{J\,k}{}_{jk}\,.$$

Volume form:

$$g_0 = \prod_{I=1}^{N}\left(g^{I}\right)^{\frac{1}{N}}$$

Parameter matrices

- Calculate linearized field equations.
- Eigenvalues of the parameter matrices:

$$P_1 = -2Q_1 = -2R_1 = -2M_1 = 2N_1 = Nx + y\,,$$
$$P_0 = -Nx + y - w + r - 2s\,, \quad R_0 = M_0 = \frac{Nx - v + 2s}{2}\,,$$
$$Q_0 = \frac{Nx - y + w - 3r}{2}\,, \quad N_0 = \frac{-Nx - y - u + v - s}{2}\,.$$

$\Rightarrow$ Field equations are gauge invariant. ✓

$\Rightarrow$ Gravitational action is diffeomorphism invariant. ✓

Parametrized post-Newtonian (PPN) formalism

- Obtain “fingerprint” of metric gravity theories. [Thorne, Will ’71; Will ’93]
  - Characterize gravity theories by 10 parameters.
  - PPN parameters can be measured by solar system experiments.
- Extension to multimetric gravity theories. [MH, M. Wohlfarth ’10]
  - Additional 14 unobserved parameters.
  - 8 parameters can be obtained from linearized field equations.

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  - Additional 14 unobserved parameters.
  - 8 parameters can be obtained from linearized field equations.
- PPN formalism applied to example theory.
- Restriction of input parameters:

$$y = \frac{1}{2-N} - Nx\,, \quad v = \frac{6-N}{4-2N} - Nx + 2u\,,$$
$$w = -\frac{6-N}{4-2N} + Nx - 3u\,, \quad r = -\frac{1}{2-N} + Nx - u\,.$$

⇒ Three free parameters x, u, s remain.

⇒ Consistent with solar system experiments up to linear PPN order.

PPN consistent multimetric example

Parameter matrices:

$$P_1 = -2Q_1 = -2R_1 = -2M_1 = 2N_1 = \frac{1}{2-N}\,,$$
$$P_0 = \frac{6-N}{4-2N} - 2Nx + 2u - 2s\,, \quad N_0 = \frac{4-N}{8-4N} + \frac{-Nx+u-s}{2}\,,$$
$$Q_0 = -\frac{1}{4}\,, \quad R_0 = M_0 = -\frac{6-N}{8-4N} + Nx - u + s\,.$$

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- Dependent only on single parameter $p := Nx - u + s$.

Generic case:

- E(2) class for $\mathfrak{h}^1$: N_2
- E(2) class for $\mathfrak{h}^i$: N_2

$\Rightarrow$ Effective E(2) class: N_2

Special case $p = \frac{6-N}{8-4N}$:

- E(2) class for $\mathfrak{h}^1$: N_2
- E(2) class for $\mathfrak{h}^i$: II_6

$\Rightarrow$ Effective E(2) class: II_6

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- Gravitational waves:
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  - Polarizations given by representations of E(2)
  - E(2) classes N_2, N_3, III_5, II_6
- Example theory:
  - Defined by gravitational action
  - Consistent up to linear PPN level
  - 3 free parameters
  - E(2) class either N_2 or II_6

Outlook

- Production of gravitational waves:
  - Slow-moving sources
  - Binary systems
    - Expansion in multipole coefficients
    - Make use of tensor solid harmonics $\Rightarrow$ algebraic equations

    $\rightsquigarrow$ Watch the arXiv!
  - Numerical simulations

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$\rightsquigarrow$ Experimental data from gravitational wave experiments!