Aspects of multimetric gravity

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DER FORSCHUNG | DER LEHRE | DER BILDUNG

3Quantum: Algebra Geometry Information
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Outline

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Motivation

- ΛCDM model: only 5% visible matter.
  - Dark matter explains “missing mass” in galaxies.
  - Dark energy explains accelerating expansion.
  - Constituents of dark universe are unknown!

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- Idea here: Additional "dark, negative mass" standard model copy.
- Only interaction between both copies: repulsive gravity.
- Universe contains equal amounts of both types of mass:

$\rightsquigarrow$ Dark galaxies "push" visible matter & light towards visible galaxies.
$\Rightarrow$ Explanation of dark matter!

$\rightsquigarrow$ Mutual repulsion between galaxies drives accelerating expansion.
$\Rightarrow$ Explanation of dark energy!

Action and equations of motion

- N metric tensors and N standard model copies.
- Matter action: sum of standard model actions.
- Gravitational action:

$$S_G[g^1, \ldots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \left[\sum_{I,J=1}^{N} (x + y\delta^{IJ}) g^{Iij} R^J_{ij} + F(S^{IJ}) \right].$$

- Symmetric volume form $g_0 = (g^1 g^2 \ldots g^N)^{1/N}$.
- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^I - \Gamma^J$.

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⇒ Equations of motion:

$$T^I_{ab} = \sqrt{\frac{g_0}{g^I}}\left[-\frac{1}{2N}g^I_{ab}\sum_{J,K=1}^{N}(x+y\delta^{JK})g^{Jij}R^K{}_{ij} + \sum_{J=1}^{N}(x+y\delta^{IJ})R^J{}_{ab}\right]$$

$$+ \text{ terms linear in } \nabla^I S^{JK}$$

$$+ \text{ terms quadratic in } S^{IJ}.$$

⇒ Repulsive Newtonian limit for $N \geq 3$. [MH, M. Wohlfarth '10]

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Simple cosmological model

- Homogeneous, isotropic FLRW universe.
- Matter content given by perfect fluid matter.
- Copernican principle: common evolution for all matter sectors.
  - $\Rightarrow$ All metrics are equal: $g^I_{ab} = g_{ab}$
  - $\Rightarrow$ All energy-momentum tensors are equal: $T^I_{ab} = T_{ab}$

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$\Rightarrow$ Equations of motion simplify for repulsive Newtonian limit:

$$(2 - N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}\,.$$

$\Rightarrow$ Negative effective gravitational constant.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),$$
$$p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right).$$

⇒ Positive matter density $\rho > 0$ requires $k = -1$ and $\dot{a}^2 < 1$.

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- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{N-2}{6}(\rho + 3p).$$

$\Rightarrow$ Acceleration must be positive for standard model matter.

Explicit solution

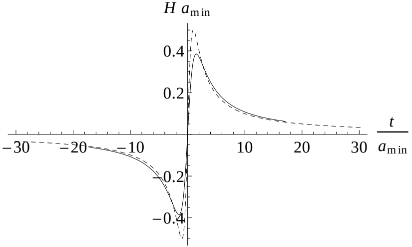
- Equation of state: $p = \omega\rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time η defined by $dt = a\,d\eta$:

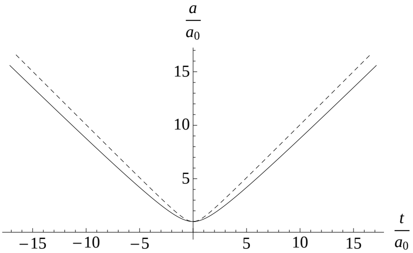
$$a = a_0 \left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{\frac{2}{3\omega+1}},$$

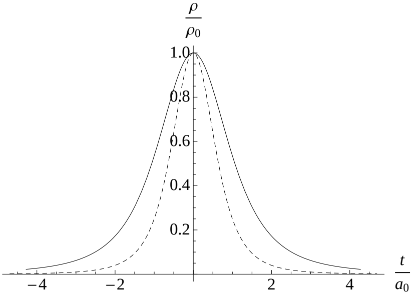
$$\rho = \frac{3}{(N-2)a_0^2}\left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{-\frac{6\omega+6}{3\omega+1}}.$$

$\Rightarrow$ Positive minimal radius a_0. [MH, M. Wohlfarth '10]

Cosmological evolution







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Structure formation

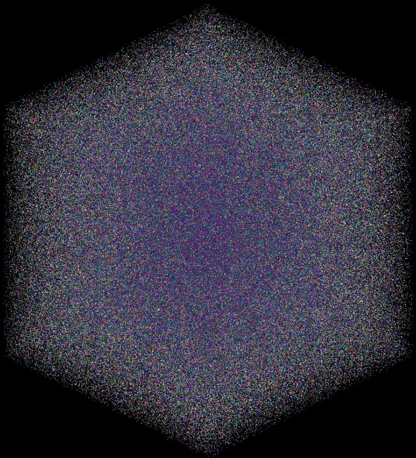
- Structure formation in ΛCDM not fully understood:
  - Missing dwarf problem. [Moore *et al.* '99]
  - Core-cusp-problem. [Dubinski, Carlberg '91; Navarro *et al.* '96]
- Structure formation in multimetric gravity:
  - Perturbation of cosmological background solution.
  - Model dust matter by point particles.
  - Interaction between point particles given by Newtonian limit.
- Implementation:
  - Large particle number requires high computing power.

  $\Rightarrow$ Use GPU computing!
- Results:
  - Galactic clusters and filament-like structures.
  - Seemingly empty voids contain "invisible" matter.

  $\Rightarrow$ Repulsive gravity effects from galactic voids.

  $\Rightarrow$ Negative gravitational lenses in galactic voids?

Structure formation - all matter types



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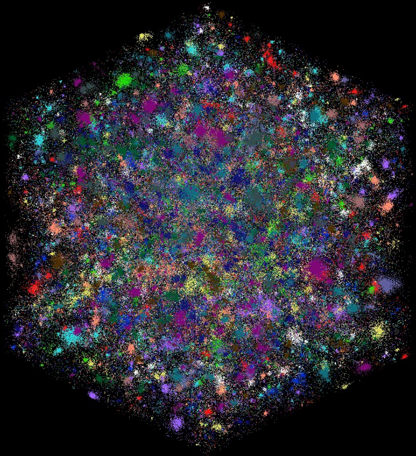
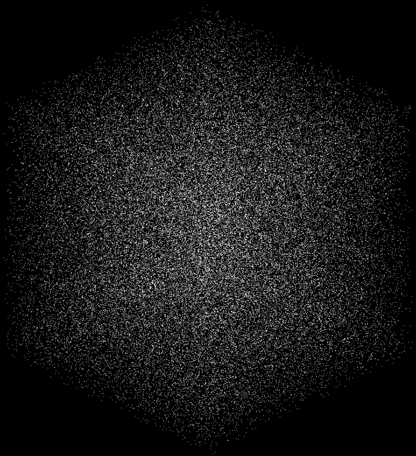


Structure formation - only visible matter



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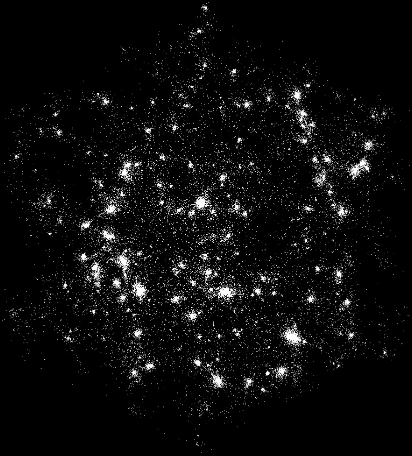

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- Obtain "fingerprint" of single-metric gravity theories. [Thorne, Will '71; Will '93]

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⇒ Additional 14 unobserved parameters.

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- Example: multimetric action can be chosen such that
  - $\alpha^+ = 1$, $\theta^+ = 0$: standard PPN gauge choice.
  - $\gamma^+ = 1$, $\sigma_+^+ = -2$: experimental consistency.
  - $\alpha^- = -1$: repulsive Newtonian limit.
  - $\gamma^- = -1$, $\theta^- = 0$, $\sigma_+^- = 2$: additional "dark" PPN parameters.

Multimetric example theory

Gravitational action with parameters x, y, u, v, w, r, s [MH '12]:

$$S_G = \frac{1}{16\pi}\int d^4x\sqrt{g_0}\left[x\sum_{I,J=1}^{N} g^{I\,ij}R^{J}{}_{ij} + \sum_{I=1}^{N} g^{I\,ij}\left(yR^{I}{}_{ij} + u\tilde{S}^{I}{}_{i}\tilde{S}^{I}{}_{j}\right.\right.$$

$$\left.\left. + v\tilde{S}^{I}{}_{k}\tilde{S}^{I\,k}{}_{ij} + w\tilde{S}^{I\,k}{}_{im}\tilde{S}^{I\,m}{}_{jk} + g^{I\,kl}g^{I}{}_{mn}\left(r\tilde{S}^{I\,m}{}_{ik}\tilde{S}^{I\,n}{}_{jl} + s\tilde{S}^{I\,m}{}_{ij}\tilde{S}^{I\,n}{}_{kl}\right)\right)\right].$$

Restriction of input parameters by PPN consistency:

$$y = \frac{1}{2-N} - Nx\,, \quad v = \frac{6-N}{4-2N} - Nx + 2u\,,$$

$$w = -\frac{6-N}{4-2N} + Nx - 3u\,, \quad r = -\frac{1}{2-N} + Nx - u\,.$$

$\Rightarrow$ PPN consistent theory with parameters x, u, s.

Outline

Linearized multimetric gravity

- Perturbation ansatz: $g^I = \eta + h^I$.
- Most general linearized vacuum field equations:

$$\underline{\underline{P}} \cdot \partial^p \partial_{(a} \underline{h}_{b)p} + \underline{\underline{Q}} \cdot \Box \underline{h}_{ab} + \underline{\underline{R}} \cdot \partial_a \partial_b \underline{h} + \underline{\underline{M}} \cdot \partial^p \partial^q \underline{h}_{pq} \eta_{ab} + \underline{\underline{N}} \cdot \Box \underline{h} \eta_{ab} = 0$$

- Parameter matrices $\underline{\underline{P}}, \underline{\underline{Q}}, \underline{\underline{R}}, \underline{\underline{M}}, \underline{\underline{N}}$.
- Diagonalize parameter matrices.

$\Rightarrow$ 10 parameters $P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0$.

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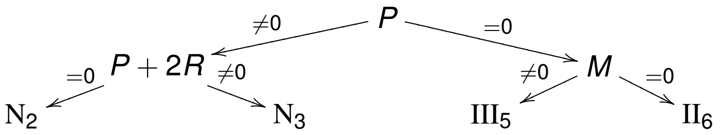
⇒ 10 parameters $P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0$.

- Calculate wave-like solutions of vacuum field equations.

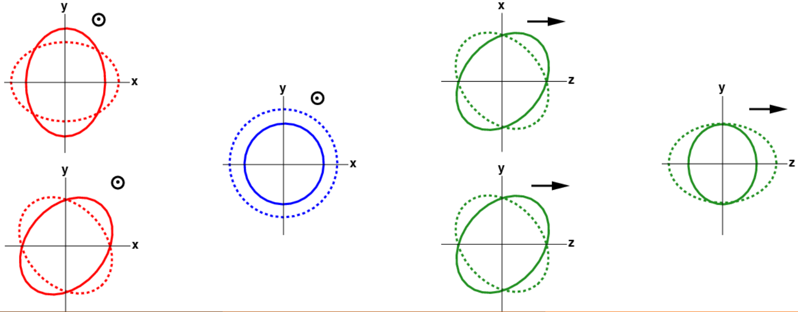
⇒ Gravitational waves propagate at the speed of light.

Polarizations and E(2) class

- Polarizations classified by reps. of E(2). [Eardley, Lee, Lightman *et al.* '73]
- E(2) class depends on parameters P_i, R_i, M_i.



2 tensors | +1 scalar | +2 vectors | +1 scalar



PPN consistent multimetric example

Parameter values [MH '12]:

$$P_1 = -2Q_1 = -2R_1 = -2M_1 = 2N_1 = \frac{1}{2-N}\,,$$

$$P_0 = \frac{6-N}{4-2N} - 2Nx + 2u - 2s\,, \quad N_0 = \frac{4-N}{8-4N} + \frac{-Nx+u-s}{2}\,,$$

$$Q_0 = -\frac{1}{4}\,, \quad R_0 = M_0 = -\frac{6-N}{8-4N} + Nx - u + s\,.$$

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- Dependent only on single parameter $p := Nx - u + s$.

Generic case:

- E(2) class for $\mathfrak{h}^1$: N_2
- E(2) class for $\mathfrak{h}^i$: N_2

$\Rightarrow$ Effective E(2) class: N_2

Special case $p = \frac{6-N}{8-4N}$:

- E(2) class for $\mathfrak{h}^1$: N_2
- E(2) class for $\mathfrak{h}^i$: II_6

$\Rightarrow$ Effective E(2) class: II_6

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Summary

- Idea: Repulsive gravity might explain dark matter & dark energy.

⇒ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.

⇒ Cosmology features late-time acceleration and big bounce.

⇒ Structure formation features clusters and voids.

⇒ Repulsive gravity is consistent with solar system experiments.

⇒ Gravitational waves are null.

⇒ E(2) class can be one of N_2, N_3, III_5, II_6.

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