Aspects of multimetric gravity

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Outline

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1. Introduction
2. Multimetric cosmology
3. Simulation of structure formation
4. Post-Newtonian consistency
5. Gravitational waves
6. Conclusion

Einstein gravity

- Gravity is described by metric tensor g_{ab}.
- Einstein-Hilbert action:

$$S_G = \frac{1}{2}\int \omega R\,.$$

  - Volume form ω.
  - Scalar curvature R.
- Minimally coupled matter action:

$$S_M = \int \omega \mathcal{L}_M\,.$$

- Einstein equations:

$$R_{ab} - \frac{1}{2}R g_{ab} = T_{ab}\,.$$

Application to the universe

- 4.6% visible matter.

  [Komatsu *et al.* ’09]

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- 22.8% dark matter.
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    [de Blok, Bosma '02]
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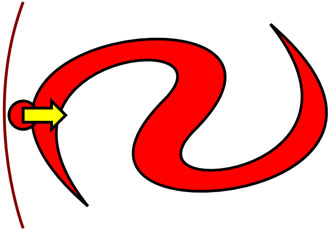

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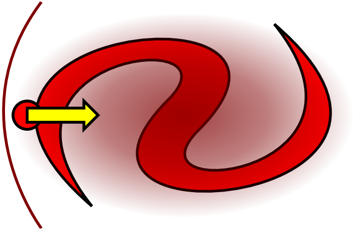

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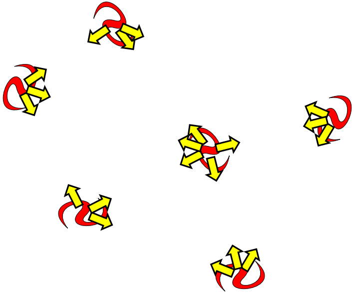

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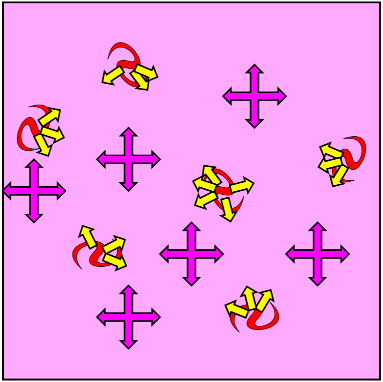

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$\Rightarrow$ Problem: What are dark matter and dark energy?

Explanations for the dark universe

- Particle physics:
  - Dark matter: [Bertone, Hooper, Silk '05]
    - Weakly interacting massive particles (WIMPs). [Ellis *et al.* '84]
    - Axions. [Preskill, Wise, Wilczek '83]
    - Massive compact halo objects (MACHOs). [Paczynski '86]
  - Dark energy: [Copeland, Sami, Tsujikawa '06]
    - Quintessence. [Peebles, Ratra '88]
    - K-essense. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
    - Chaplygin gas. [Kamenshchik, Moschella, Pasquier '01]
- Gravity:
  - Modified Newtonian dynamics (MOND). [Milgrom '83]
  - Tensor-vector-scalar theories. [Bekenstein '04]
  - Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
  - Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
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  - New idea: repulsive gravity $\Leftrightarrow$ negative mass!

Mass in Newtonian gravity

- Three types of mass! [Bondi '57]
  - Active gravitational mass m_a - source of gravity: $\phi = -G_N \frac{m_a}{r}$.
  - Passive gravitational mass m_p - reaction on gravity: $\vec{F} = -m_p \vec{\nabla} \phi$.
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- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.
- Observations exist for visible mass only.

Dark universe from negative mass

- Idea for dark universe: standard model with $m_a = m_p = -m_i < 0$.
- Both copies couple only through gravity $\Rightarrow$ “dark”.
- Preserves momentum conservation.
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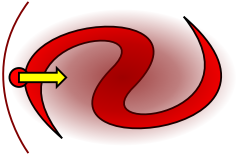

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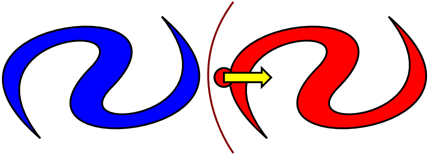

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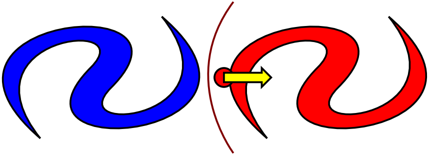

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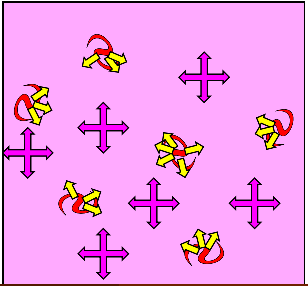

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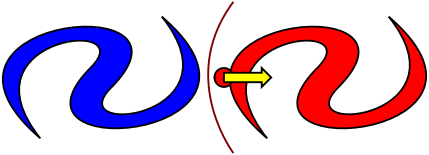

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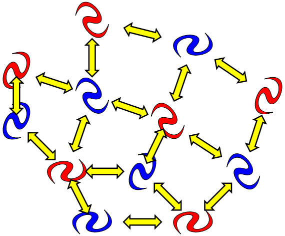

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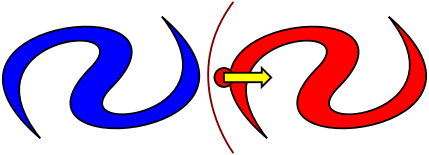

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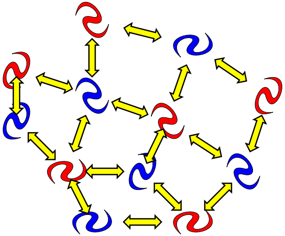
$\Rightarrow$ Advantage: Dark copy Ψ^- of well-known standard model Ψ^+:

  - No new parameters.
  - No unknown masses.
  - No unknown couplings.



Repulsive Einstein gravity

- Positive and negative test masses follow different trajectories.
- Two types of test mass trajectories $\Rightarrow$ two types of observers.
- Observer trajectories are autoparallels of two connections $\nabla^{\pm}$.
- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.

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- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.
- No-go theorem forbids bimetric repulsive gravity. [MH, M. Wohlfarth '09]
- Solution: $N \geq 3$ metrics g^{I}_{ab} and standard model copies Ψ^{I}.

Action and equations of motion

- N metric tensors and N standard model copies.
- Matter action: sum of standard model actions.
- Gravitational action:

$$S_G[g^1, \ldots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \left[\sum_{I,J=1}^{N} (x + y\delta^{IJ}) g^{Iij} R^J_{ij} + F(S^{IJ}) \right].$$

- Symmetric volume form $g_0 = (g^1 g^2 \ldots g^N)^{1/N}$.
- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^I - \Gamma^J$.

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$\Rightarrow$ Equations of motion:

$$T^I_{ab} = \sqrt{\frac{g_0}{g^I}}\left[-\frac{1}{2N}g^I_{ab}\sum_{J,K=1}^{N}(x + y\delta^{JK})g^{Jij}R^K{}_{ij} + \sum_{J=1}^{N}(x + y\delta^{IJ})R^J{}_{ab}\right]$$

$$+ \text{ terms linear in } \nabla^I S^{JK}$$

$$+ \text{ terms quadratic in } S^{IJ}\,.$$

$\Rightarrow$ Repulsive Newtonian limit for $N \geq 3$. [MH, M. Wohlfarth '10]

Outline

Cosmological symmetry

- Standard cosmology: Robertson–Walker metrics

$$g^I = -n_I^2(t)dt \otimes dt + a_I^2(t)\gamma_{\alpha\beta}dx^\alpha \otimes dx^\beta\,.$$

  - Lapse functions n_I.
  - Scale factors a_I.
  - Spatial metric $\gamma_{\alpha\beta}$ of constant curvature $k \in \{-1, 0, 1\}$ and Riemann tensor $R(\gamma)_{\alpha\beta\gamma\delta} = 2k\gamma_{\alpha[\gamma}\gamma_{\delta]\beta}$.

- Perfect fluid matter:

$$T^{I\,ab} = (\rho_I + p_I)u^{Ia}u^{Ib} + p_I g^{Iab}\,.$$

- Normalization: $g^I_{ab}u^{Ia}u^{Ib} = -1$.

Simple cosmological model

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$\Rightarrow$ Single effective energy-momentum tensor $T^I_{ab} = T_{ab}$.

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$\Rightarrow$ Common scale factors $a^I = a$ and lapse functions $n^I = n$.

$\Rightarrow$ Rescale cosmological time to set $n \equiv 1$.

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$\Rightarrow$ Equations of motion simplify:

$$(2 - N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}\,.$$

$\Rightarrow$ Negative effective gravitational constant for early / late universe.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),$$
$$p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right).$$

$\Rightarrow$ Positive matter density $\rho > 0$ requires $k = -1$ and $\dot{a}^2 < 1$.

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- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{N-2}{6}(\rho + 3p).$$

$\Rightarrow$ Acceleration must be positive for standard model matter.

Explicit solution

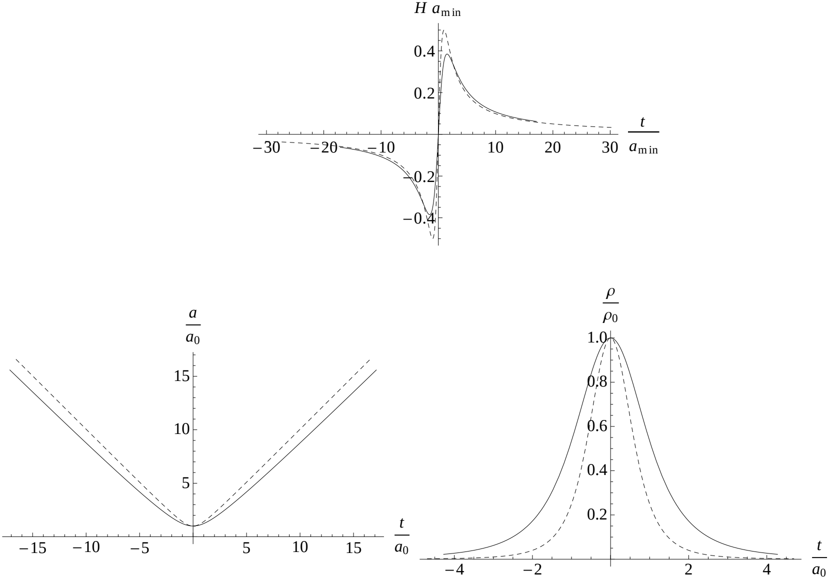
- Equation of state: $p = \omega\rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time η defined by $dt = a\, d\eta$:

$$a = a_0 \left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{\frac{2}{3\omega+1}},$$

$$\rho = \frac{3}{(N-2)a_0^2}\left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{-\frac{6\omega+6}{3\omega+1}}.$$

$\Rightarrow$ Positive minimal radius a_0. [MH, M. Wohlfarth '10]

Cosmological evolution



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Structure formation

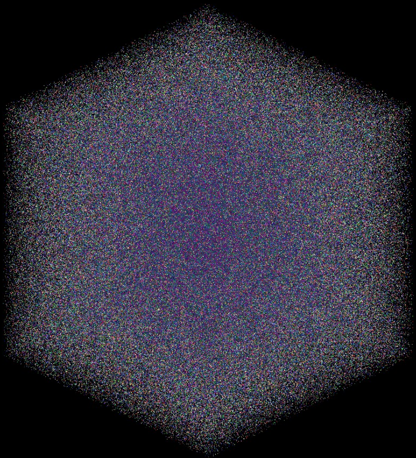
- Formation of galactic structures not fully understood:
  - Missing dwarf problem. [Moore *et al.* '99]
  - Core-cusp-problem. [Dubinski, Carlberg '91; Navarro *et al.* '96]
- Structure formation in multimetric gravity:
  - Perturbation of cosmological background solution.
  - Model dust matter by point particles.
  - Interaction between point particles given by Newtonian limit.
- Implementation:
  - Large particle number requires high computing power.

  $\Rightarrow$ Use GPU computing!
- Results:
  - Galactic clusters and filament-like structures.
  - Seemingly empty voids contain “invisible” matter.

  $\Rightarrow$ Repulsive gravity effects from galactic voids.

  $\Rightarrow$ Negative gravitational lenses in galactic voids?

Structure formation - all matter types



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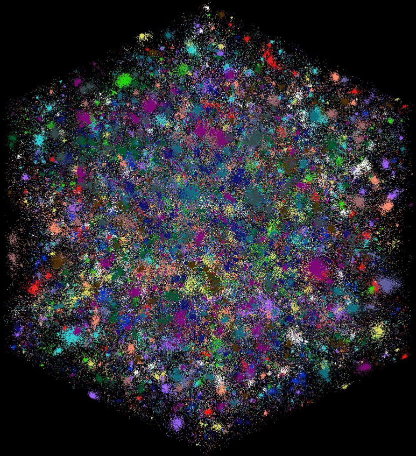
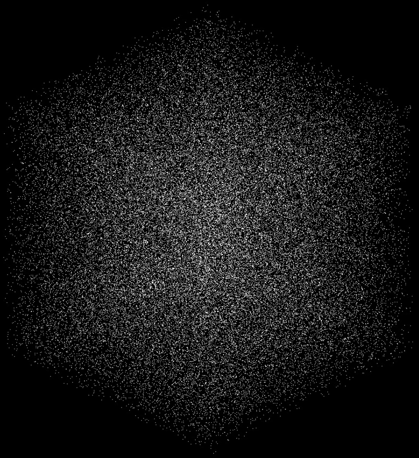


Structure formation - only visible matter



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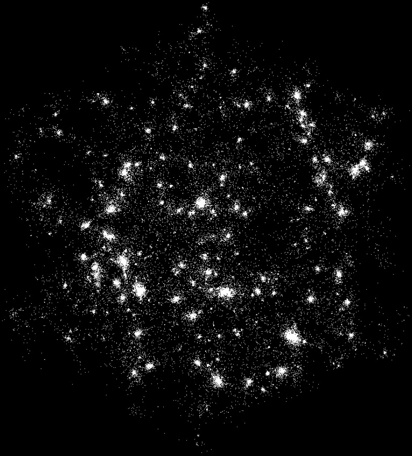

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Parametrized post-Newtonian formalism

- Obtain "fingerprint" of single-metric gravity theories. [Thorne, Will '71; Will '93]

$\Rightarrow$ 10 parameters, constrained by solar system experiments.

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- Example: multimetric action can be chosen such that
  - $\alpha^+ = 1$, $\theta^+ = 0$: standard PPN gauge choice.
  - $\gamma^+ = 1$, $\sigma_+^+ = -2$: experimental consistency.
  - $\alpha^- = -1$: repulsive Newtonian limit.
  - $\gamma^- = -1$, $\theta^- = 0$, $\sigma_+^- = 2$: additional "dark" PPN parameters.

Multimetric example theory

Gravitational action with parameters x, y, u, v, w, r, s [MH '12]:

$$S_G = \frac{1}{16\pi}\int d^4x\sqrt{g_0}\left[x\sum_{I,J=1}^{N} g^{I\,ij}R^J{}_{ij} + \sum_{I=1}^{N} g^{I\,ij}\left(yR^I{}_{ij} + u\tilde{S}^I{}_i\tilde{S}^I{}_j\right.\right.$$

$$\left.\left. + v\tilde{S}^I{}_k\tilde{S}^{I\,k}{}_{ij} + w\tilde{S}^{I\,k}{}_{im}\tilde{S}^{I\,m}{}_{jk} + g^{I\,kl}g^I{}_{mn}\left(r\tilde{S}^{I\,m}{}_{ik}\tilde{S}^{I\,n}{}_{jl} + s\tilde{S}^{I\,m}{}_{ij}\tilde{S}^{I\,n}{}_{kl}\right)\right)\right].$$

Restriction of input parameters by PPN consistency:

$$y = \frac{1}{2-N} - Nx\,, \quad v = \frac{6-N}{4-2N} - Nx + 2u\,,$$
$$w = -\frac{6-N}{4-2N} + Nx - 3u\,, \quad r = -\frac{1}{2-N} + Nx - u\,.$$

$\Rightarrow$ PPN consistent theory with parameters x, u, s.

Outline

Linearized multimetric gravity

- Perturbation ansatz: $g^I = \eta + h^I$.
- Most general linearized vacuum field equations:

$$\underline{\underline{P}} \cdot \partial^p \partial_{(a} \underline{h}_{b)p} + \underline{\underline{Q}} \cdot \Box \underline{h}_{ab} + \underline{\underline{R}} \cdot \partial_a \partial_b \underline{h} + \underline{\underline{M}} \cdot \partial^p \partial^q \underline{h}_{pq} \eta_{ab} + \underline{\underline{N}} \cdot \Box \underline{h} \eta_{ab} = 0$$

- Parameter matrices $\underline{\underline{P}}, \underline{\underline{Q}}, \underline{\underline{R}}, \underline{\underline{M}}, \underline{\underline{N}}$.
- Diagonalize parameter matrices.

$\Rightarrow$ 10 parameters $P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0$.

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- Perturbation ansatz: $g^I = \eta + h^I$.
- Most general linearized vacuum field equations:

$$\underline{\underline{P}} \cdot \partial^p \partial_{(a} \underline{h}_{b)p} + \underline{\underline{Q}} \cdot \Box \underline{h}_{ab} + \underline{\underline{R}} \cdot \partial_a \partial_b \underline{h} + \underline{\underline{M}} \cdot \partial^p \partial^q \underline{h}_{pq} \eta_{ab} + \underline{\underline{N}} \cdot \Box \underline{h} \eta_{ab} = 0$$

- Parameter matrices $\underline{\underline{P}}, \underline{\underline{Q}}, \underline{\underline{R}}, \underline{\underline{M}}, \underline{\underline{N}}$.
- Diagonalize parameter matrices.

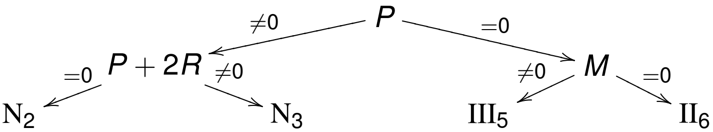
$\Rightarrow$ 10 parameters $P_1, P_0, Q_1, Q_0, R_1, R_0, M_1, M_0, N_1, N_0$.

- Calculate wave-like solutions of vacuum field equations.

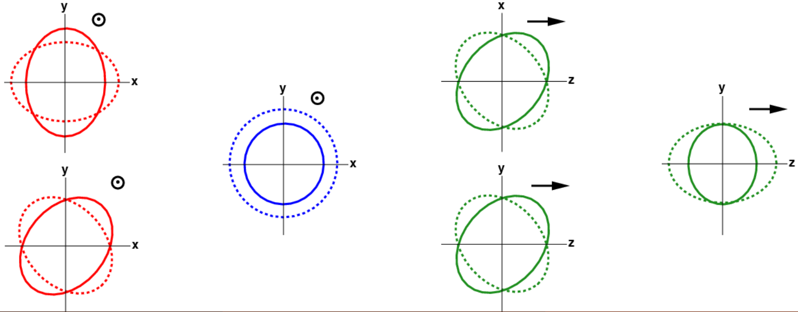
$\Rightarrow$ Gravitational waves propagate at the speed of light.

Polarizations and E(2) class

- Polarizations classified by reps. of E(2). [Eardley, Lee, Lightman *et al.* '73]
- E(2) class depends on parameters P_i, R_i, M_i.



2 tensors | +1 scalar | +2 vectors | +1 scalar



PPN consistent multimetric example

Parameter values [MH '12]:

$$P_1 = -2Q_1 = -2R_1 = -2M_1 = 2N_1 = \frac{1}{2-N}\,,$$

$$P_0 = \frac{6-N}{4-2N} - 2Nx + 2u - 2s\,, \quad N_0 = \frac{4-N}{8-4N} + \frac{-Nx+u-s}{2}\,,$$

$$Q_0 = -\frac{1}{4}\,, \quad R_0 = M_0 = -\frac{6-N}{8-4N} + Nx - u + s\,.$$

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- Dependent only on single parameter $p := Nx - u + s$.

Generic case:

- E(2) class for $\mathfrak{h}^1$: N_2
- E(2) class for $\mathfrak{h}^i$: N_2

$\Rightarrow$ Effective E(2) class: N_2

Special case $p = \frac{6-N}{8-4N}$:

- E(2) class for $\mathfrak{h}^1$: N_2
- E(2) class for $\mathfrak{h}^i$: II_6

$\Rightarrow$ Effective E(2) class: II_6

Outline

Summary

- Idea: Repulsive gravity might explain dark matter & dark energy.

⇒ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.

⇒ Cosmology features late-time acceleration and big bounce.

⇒ Structure formation features clusters and voids.

⇒ Repulsive gravity is consistent with solar system experiments.

⇒ Gravitational waves are null.

⇒ E(2) class can be one of N_2, N_3, III_5, II_6.

Outlook

- Work in progress:
  - Emission of gravitational waves from binary systems.
  - Post-Newtonian approximation of axially symmetric solutions.

Outlook

- Work in progress:
  - Emission of gravitational waves from binary systems.
  - Post-Newtonian approximation of axially symmetric solutions.
- Future work:
  - Remaining PPN parameters from full multimetric PPN formalism.
  - Restrict multimetric gravity theories by additional PPN bounds.
  - Further construction principles, e.g., higher symmetries.
  - Construct further exact solutions.
  - Stabillity of cosmological solutions.
  - Obtain restrictions from cosmological perturbation theory.