Multimetric cosmology and structure formation

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Outline

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Einstein gravity

- Gravity is described by metric tensor g_{ab}.
- Einstein-Hilbert action:

$$S_G = \frac{1}{2} \int \omega R \,.$$

  - Volume form ω.
  - Scalar curvature R.
- Minimally coupled matter action:

$$S_M = \int \omega \mathcal{L}_M \,.$$

- Einstein equations:

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} \,.$$

Application to the universe

- 4.6% visible matter.

  [Komatsu *et al.* '09]

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  - Galaxy rotation curves.

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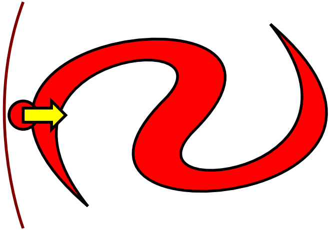

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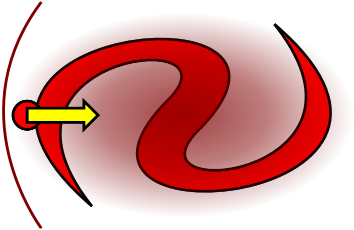

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  - Accelerating expansion.

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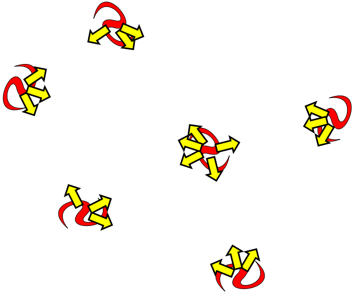

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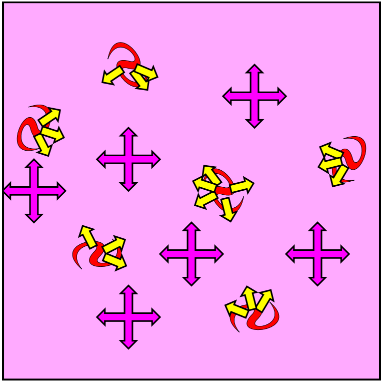

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- 22.8% dark matter?
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$\Rightarrow$ Problem: What are dark matter and dark energy?

Explanations for the dark universe

- Particle physics:
  - Dark matter: [Bertone, Hooper, Silk '05]
    - Weakly interacting massive particles (WIMPs). [Ellis *et al.* '84]
    - Axions. [Preskill, Wise, Wilczek '83]
    - Massive compact halo objects (MACHOs). [Paczynski '86]
  - Dark energy: [Copeland, Sami, Tsujikawa '06]
    - Quintessence. [Peebles, Ratra '88]
    - K-essense. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
    - Chaplygin gas. [Kamenshchik, Moschella, Pasquier '01]
- Gravity:
  - Modified Newtonian dynamics (MOND). [Milgrom '83]
  - Tensor-vector-scalar theories. [Bekenstein '04]
  - Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
  - Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
  - Non-symmetric gravity. [Moffat '95]
  - Area metric gravity. [Punzi, Schuller, Wohlfarth '07]

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  - New idea: repulsive gravity $\Leftrightarrow$ negative mass!

Mass in Newtonian gravity

- Three types of mass! [Bondi '57]
  - Active gravitational mass m_a - source of gravity: $\phi = -G_N \frac{m_a}{r}$.
  - Passive gravitational mass m_p - reaction on gravity: $\vec{F} = -m_p \vec{\nabla}\phi$.
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- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.
- Observations exist for visible mass only.

Dark universe from negative mass

- Idea for dark universe: standard model with $m_a = m_p = -m_i < 0$.
- Both copies couple only through gravity $\Rightarrow$ “dark”.
- Preserves momentum conservation.
- Breaks weak equivalence principle only for cross-interaction.

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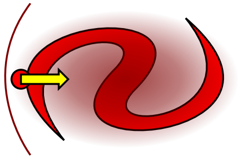

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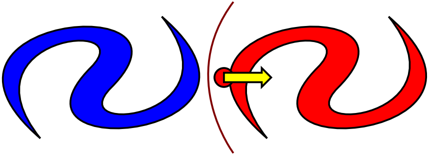

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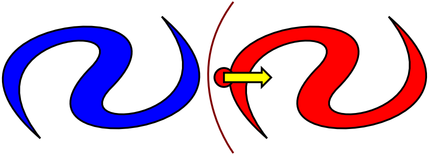

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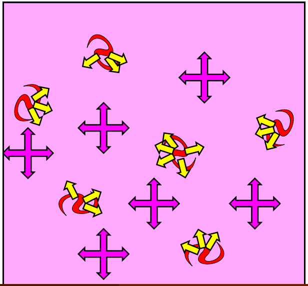

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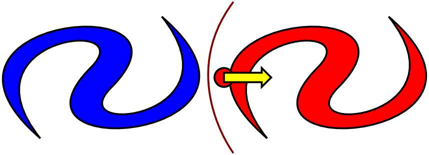

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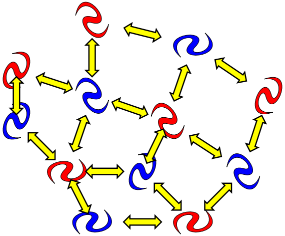

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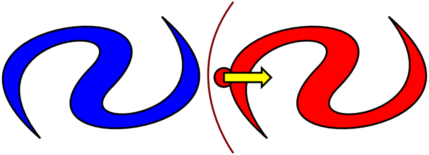
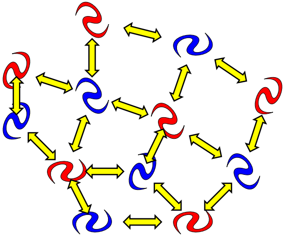


- Explanation of dark energy.

$\Rightarrow$ Advantage: Dark copy Ψ^- of well-known standard model Ψ^+:
  - No new parameters.
  - No unknown masses.
  - No unknown couplings.



Repulsive Einstein gravity

- Positive and negative test masses follow different trajectories.
- Two types of test mass trajectories $\Rightarrow$ two types of observers.
- Observer trajectories are autoparallels of two connections $\nabla^{\pm}$.
- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.

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- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.
- No-go theorem forbids bimetric repulsive gravity. [MH, M. Wohlfarth '09]
- Solution: $N \geq 3$ metrics g^{I}_{ab} and standard model copies Ψ^{I}.

Action and equations of motion

- N metric tensors and N standard model copies.
- Matter action: sum of standard model actions.
- Gravitational action:

$$S_G[g^1, \ldots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \left[\sum_{I,J=1}^{N} c^{IJ} g^{Iij} R^J_{ij} + F(S^{IJ}) \right] .$$

- Symmetric volume form $g_0 = (g^1 g^2 \ldots g^N)^{1/N}$.
- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^I - \Gamma^J$.

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⇒ Equations of motion:

$$T^I_{ab} = \sqrt{\frac{g_0}{g^I}} \left[-\frac{1}{2N} g^I_{ab} \sum_{J,K=1}^{N} c^{JK} g^{Jij} R^K{}_{ij} + \sum_{J=1}^{N} c^{IJ} R^J{}_{ab} \right]$$

$$+ \text{ terms linear in } \nabla^I S^{JK}$$

$$+ \text{ terms quadratic in } S^{IJ} .$$

⇒ Repulsive Newtonian limit for $N \geq 3$. [MH, M. Wohlfarth '10]

Outline

Cosmological symmetry

- Standard cosmology: Robertson–Walker metrics

$$g^I = -n_I^2(t)dt \otimes dt + a_I^2(t)\gamma_{\alpha\beta}dx^\alpha \otimes dx^\beta\,.$$

  - Lapse functions n_I.
  - Scale factors a_I.
  - Spatial metric $\gamma_{\alpha\beta}$ of constant curvature $k \in \{-1, 0, 1\}$ and Riemann tensor $R(\gamma)_{\alpha\beta\gamma\delta} = 2k\gamma_{\alpha[\gamma}\gamma_{\delta]\beta}$.
- Perfect fluid matter:

$$T^{I\,ab} = (\rho_I + p_I)u^{Ia}u^{Ib} + p_I g^{Iab}\,.$$

- Normalization: $g^I_{ab}u^{Ia}u^{Ib} = -1$.

Simple cosmological model

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- Copernican principle: common evolution for all matter sectors.

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$\Rightarrow$ Single effective energy-momentum tensor $T^I_{ab} = T_{ab}$.

$\Rightarrow$ Single effective metric $g^I_{ab} = g_{ab}$.

$\Rightarrow$ Common scale factors $a^I = a$ and lapse functions $n^I = n$.

$\Rightarrow$ Rescale cosmological time to set $n \equiv 1$.

$\Rightarrow$ Ricci tensors $R^I_{ab} = R_{ab}$ become equal.

$\Rightarrow$ Connection differences $S^{IJ\,i}{}_{jk} = 0$ vanish.

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⇒ Ricci tensors $R^I_{ab} = R_{ab}$ become equal.

⇒ Connection differences $S^{IJ\,i}{}_{jk} = 0$ vanish.

⇒ Equations of motion simplify for repulsive Newtonian limit:

$$(2 - N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}\,.$$

⇒ Negative effective gravitational constant for early / late universe.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),$$
$$p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right).$$

$\Rightarrow$ Positive matter density $\rho > 0$ requires $k = -1$ and $\dot{a}^2 < 1$.

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- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{N-2}{6}(\rho + 3p).$$

$\Rightarrow$ Acceleration must be positive for standard model matter.

Explicit solution

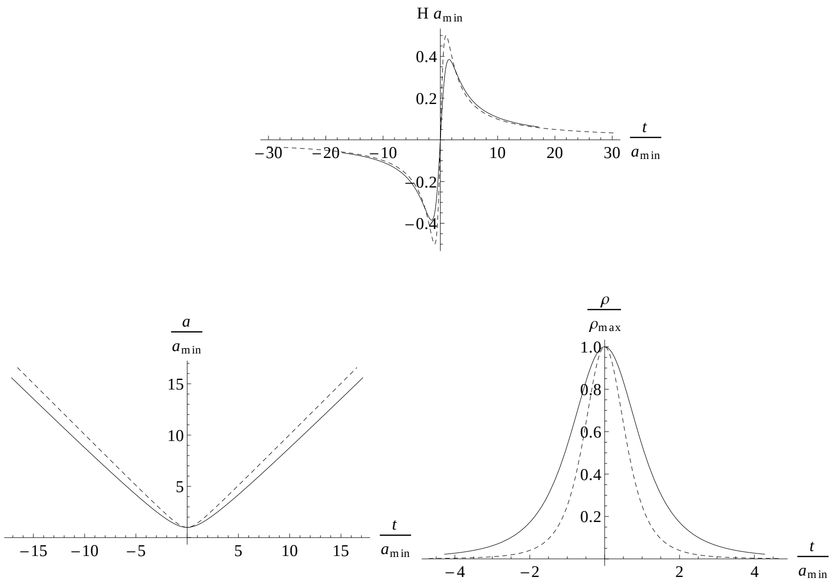
- Equation of state: $p = \omega\rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time η defined by $dt = a\,d\eta$:

$$a = a_{\text{min}} \left(\cosh\left(\frac{3\omega + 1}{2}\eta\right)\right)^{\frac{2}{3\omega+1}},$$

$$\rho = \frac{3}{(N-2)a_{\text{min}}^2} \left(\cosh\left(\frac{3\omega + 1}{2}\eta\right)\right)^{-\frac{6\omega+6}{3\omega+1}}.$$

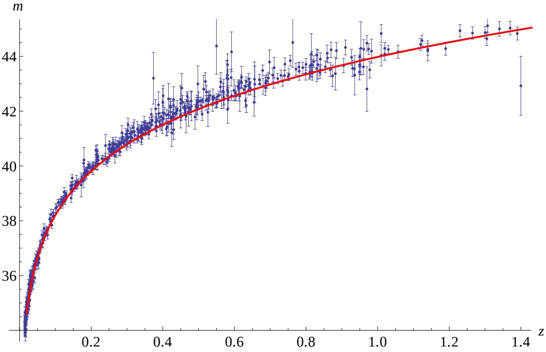
$\Rightarrow$ Positive minimal radius a_{min} (Big Bounce). [MH, M. Wohlfarth '10]

Cosmological evolution



Cosmological parameters

- Friedmann equation: $(2 - N)\Omega_M + \Omega_K = 1$.
- Matter density:
$$\Omega_M = \frac{\rho_0}{3H_0^2} \sim \sinh^{-2}\left(\frac{3\omega + 1}{2}\eta_0\right).$$
- Curvature parameter:
$$\Omega_K = -\frac{k}{a_0^2 H_0^2} = \frac{1}{\dot{a}^2(t_0)} \to 1\,.$$
- Fitting of supernova data: [Amanullah *et al.* '10]



Outline

Ingredients

- Metrics $g^I_{ab} = g^0_{ab} + h^I_{ab}$ with
$$g^0 = -dt \otimes dt + a^2(t)\gamma_{\alpha\beta}dx^\alpha \otimes dx^\beta$$
and $a(t)$ determined by cosmology.
- Scale for structure formation $\ll$ curvature radius of the universe:
  - Cubic volume $0 \leq x^\alpha \leq \ell$.
  - Approximate $\gamma_{\alpha\beta}$ by $\delta_{\alpha\beta}$.
  - Periodic boundary conditions.
- Matter content: n point masses M for each sector.
  - Model for dust matter: $p = 0$.
  - Matter density:
$$\rho = \frac{Mn}{(a\ell)^3}\,.$$
- Large mean distance $a\ell/\sqrt[3]{Nn} \gg 2GM$.
- Small peculiar velocities $|v^\alpha_{Ii}| = |a\dot{x}^\alpha_{Ii}| \ll 1$.

Local dynamics

- Masses of type I follow geodesics of their metric g^I_{ab}:

$$\ddot{x}^\alpha_{Ii} = \frac{\partial_\alpha h^I_{00}}{2a^2} - 2\frac{\dot{a}}{a}\dot{x}^\alpha_{Ii}\,.$$

- Antisymmetric Poisson equation:

$$h^I_{00} = -2\sum_{J=1}^{N}(2\delta^{IJ} - 1)\Phi^J\,.$$

- Individual Newtonian potentials $\Phi^I(t, \vec{x})$:

$$\Phi^I(t, \vec{x}) = -\frac{M}{a(t)}\sum_{i=1}^{n}\frac{1}{d(\vec{x}, \vec{x}_{Ii}(t))}\,.$$

- Periodic distance function $d(\vec{x}, \vec{x'})$:

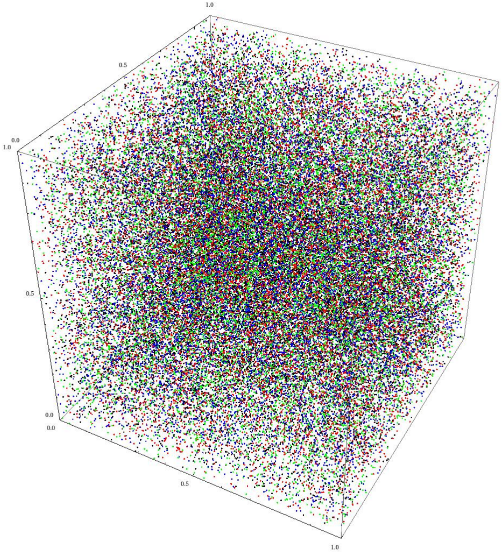
$$d(\vec{x}, \vec{x'}) = \min_{\vec{k}\in\mathbb{Z}^3}\left|\vec{x} - \vec{x}' + \ell\vec{k}\right|\,.$$

Implementation

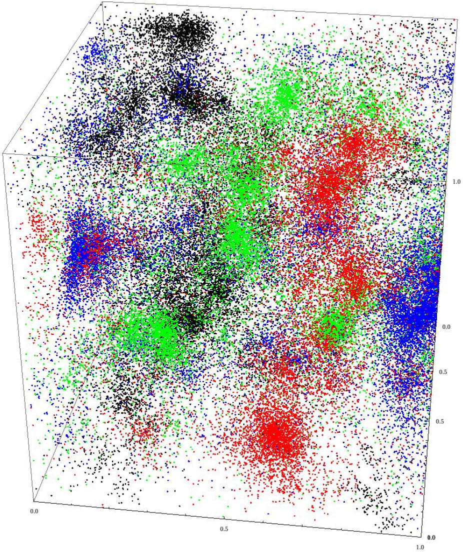
- $N = 4$ matter types.
- $n = 16384$ point masses for each matter type.
- 2000 calculation steps.
- Simulation written in C.
- Calculation using 3.0 GHz Intel Core 2 Duo E8400 CPU.

$\Rightarrow$ 7.5 days computation time.

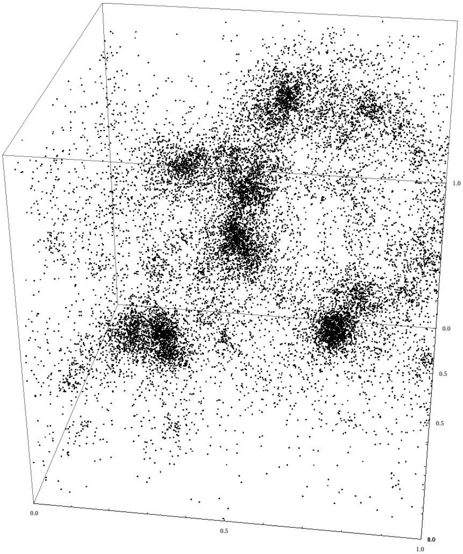
Evolution - all matter types ($N = 4\,, n = 16384$)



Final state - all matter types ($N = 4\,, n = 16384$)



Final state - only visible matter ($N = 4\,, n = 16384$)



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⇒ Higher computation power required.

- Simulation written in CUDA.
- Calculation using NVidia Tesla C2075 GPU.

⇒ 2 months computation time.

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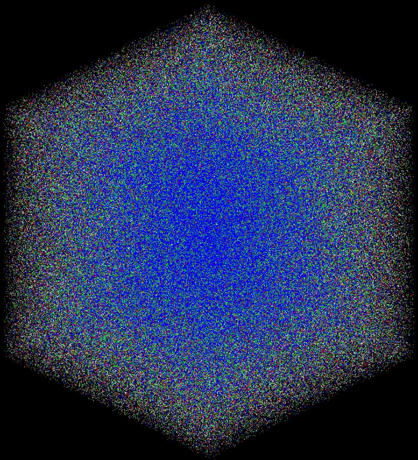

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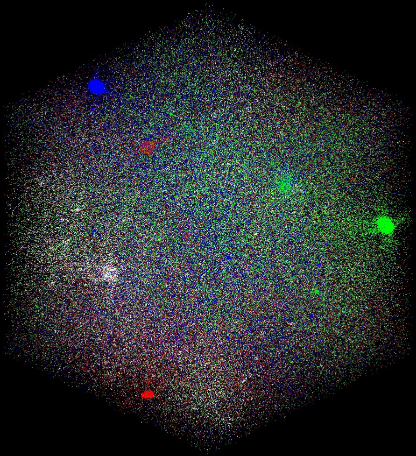

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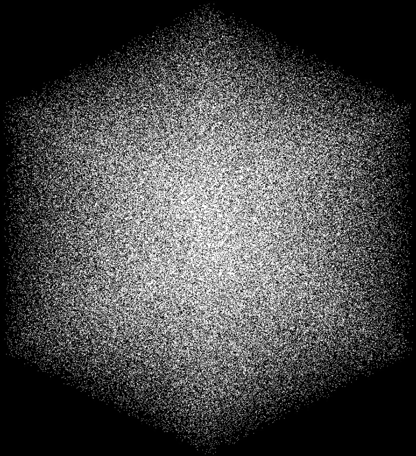
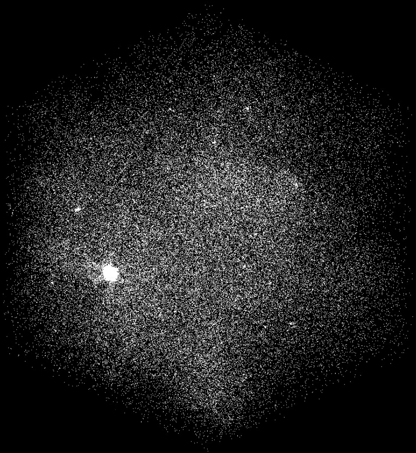


Final state - only visible matter ($N = 4$, $n = 262144$)



Intermediate result

- Structures are still very coarse.
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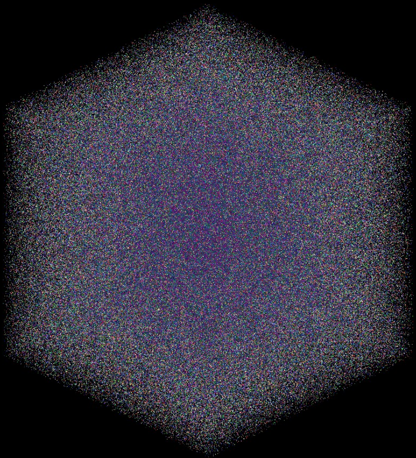
- Voids are not very empty.
- Violent dynamics.

$\Rightarrow$ Increase number of matter types.

- $N = 16$ matter types.
- $n = 65536$ point masses for each matter type.
- 31600 calculation steps.
- Simulation written in CUDA.
- Calculation using NVidia Tesla C2075 GPU.

$\Rightarrow$ 3.5 months computation time.

Evolution - all matter types ($N = 16$, $n = 65536$)



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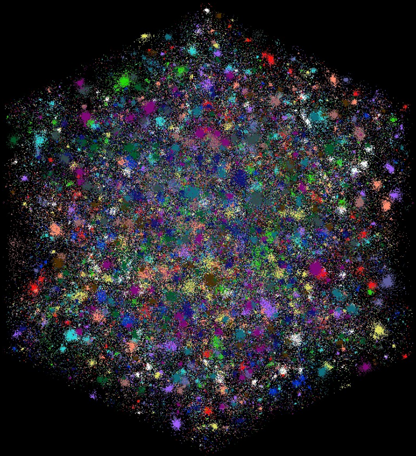

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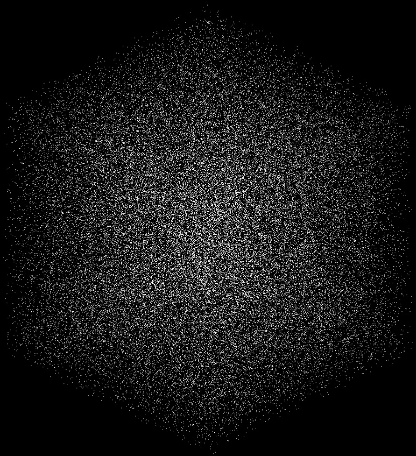

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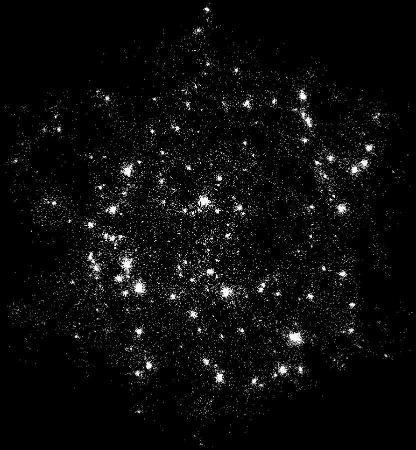

Results

- Finer structures.
- Filaments between galactic clusters.
- Large voids free of visible matter.
- Voids contain clusters of repulsively interacting matter.
  - Possible explanation for local velocity anomaly? [Tully '07]
  - Important contribution to weak lensing.
  - Strong negative gravitational lenses?

  $\Rightarrow$ Calculate gravitational lensing from simulation data!

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  $\Rightarrow$ Calculate gravitational lensing from simulation data!
- Further increase number of point masses used in the simulation?
- Problem: Currently used algorithm scales as $\mathcal{O}(n^2)$.

$\Rightarrow$ Use different algorithm!

Future work

- Adapt GADGED-2 code [Springel '05] to multimetric gravity.
- TreeSPH algorithm:
  - Gravitational forces from hierarchical multipole expansion.
  - Gas dynamics from smoothed particle hydrodynamics (SPH).
- Better scaling behavior $\mathcal{O}(n \log n)$.
- Usable on multicore PCs & clusters.

Outline

Summary

- Idea: Repulsive gravity might explain dark matter & dark energy.

⇒ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.

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Summary

- Idea: Repulsive gravity might explain dark matter & dark energy.

$\Rightarrow$ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.

- Cosmology:
  - $\Rightarrow$ Big Bounce cosmology.
  - $\Rightarrow$ Accelerating expansion becomes small at late times.
- Structure formation:
  - $\Rightarrow$ Result highly depends on number N of matter types.
  - $\Rightarrow$ Formation of galactic clusters.
  - $\Rightarrow$ Voids contain repulsively interacting, invisible matter.

Outlook

- Further cosmological calculations:
  - Analyze stability of cosmological solutions.
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- Further cosmological calculations:
  - Analyze stability of cosmological solutions.
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- Improved simulations of structure formation:
  - Use GADGED-2 code for multimetric structure formation.
  - Include thermodynamics into simulation.
- Connection to observations:
  - Test multimetric cosmology against CMB fluctuations.
  - Peculiar motion of galaxies due to repulsive matter?
  - Repulsive matter distribution in voids from weak lensing?
  - Search for negative gravitational lenses.