

Multimetric cosmology and structure formation

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21. November 2012

- 1 Introduction
- 2 Multimetric cosmology
- 3 Simulation of structure formation
- 4 Conclusion

Outline

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Einstein gravity

- Gravity is described by metric tensor g_{ab} .
- Einstein-Hilbert action:

$$S_G = \frac{1}{2} \int \omega R.$$

- Volume form ω .
 - Scalar curvature R .
- Minimally coupled matter action:

$$S_M = \int \omega \mathcal{L}_M.$$

- Einstein equations:

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab}.$$

Application to the universe

- 4.6% visible matter.

[Komatsu *et al.* '09]

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- 22.8% dark matter.

- Galaxy rotation curves.

[de Blok, Bosma '02]

- Anomalous light deflection.

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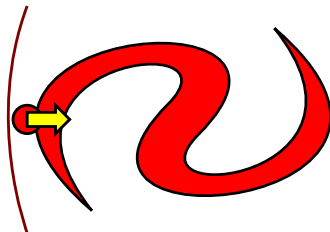
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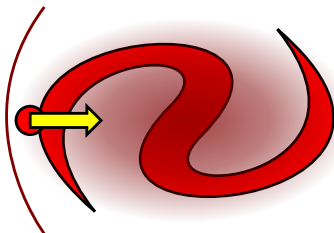
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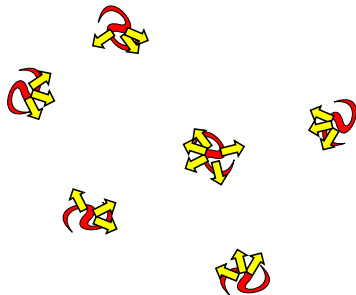
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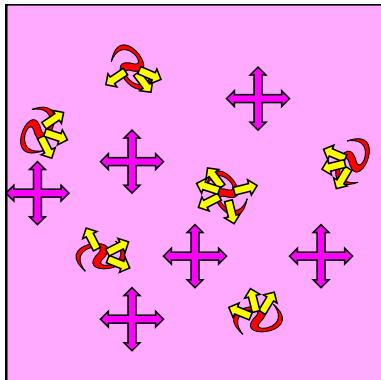
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⇒ Problem: What are dark matter and dark energy?

Explanations for the dark universe

- Particle physics:
 - Dark matter: [Bertone, Hooper, Silk '05]
 - Weakly interacting massive particles (WIMPs). [Ellis *et al.* '84]
 - Axions. [Preskill, Wise, Wilczek '83]
 - Massive compact halo objects (MACHOs). [Paczynski '86]
 - Dark energy: [Copeland, Sami, Tsujikawa '06]
 - Quintessence. [Peebles, Ratra '88]
 - K-essence. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
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- Gravity:

- Modified Newtonian dynamics (MOND). [Milgrom '83]
- Tensor-vector-scalar theories. [Bekenstein '04]
- Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
- Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
- Non-symmetric gravity. [Moffat '95]
- Area metric gravity. [Punzi, Schuller, Wohlfarth '07]

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- **New idea: repulsive gravity $\Leftrightarrow$ negative mass!**

Mass in Newtonian gravity

- Three types of mass! [Bondi '57]
 - Active gravitational mass m_a - source of gravity: $\phi = -G_N \frac{m_a}{r}$.
 - Passive gravitational mass m_p - reaction on gravity: $\vec{F} = -m_p \vec{\nabla} \phi$.
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- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.

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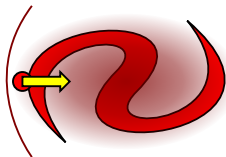
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- Observations exist for visible mass only.

Dark universe from negative mass

- Idea for dark universe: standard model with $m_a = m_p = -m_i < 0$.
- Both copies couple only through gravity $\Rightarrow$ “dark”.
- Preserves momentum conservation.
- Breaks weak equivalence principle only for cross-interaction.

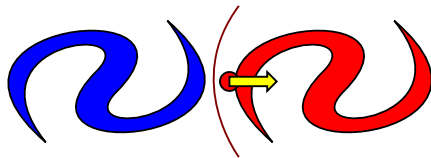
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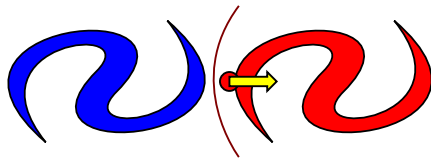
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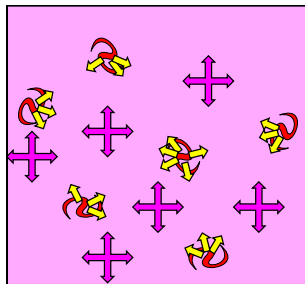


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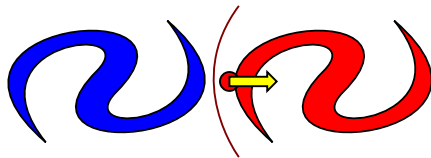


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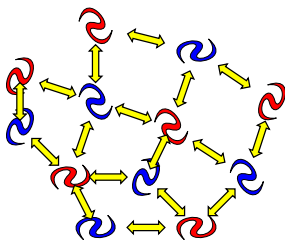


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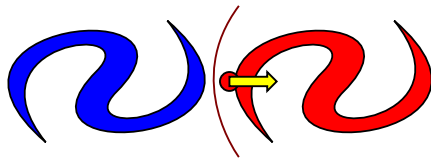


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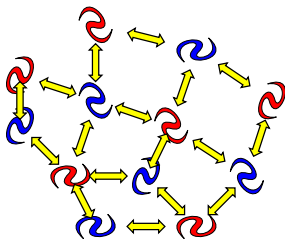
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- Explanation of dark energy.

$\Rightarrow$ Advantage: Dark copy Ψ^- of well-known standard model Ψ^+ :

- No new parameters.
- No unknown masses.
- No unknown couplings.



Repulsive Einstein gravity

- Positive and negative test masses follow different trajectories.
- Two types of test mass trajectories $\Rightarrow$ two types of observers.
- Observer trajectories are autoparallels of two connections $\nabla^\pm$.
- Observers attach parallelly transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g_{ab}^\pm$.

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- Observers attach parallelly transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g_{ab}^\pm$.
- No-go theorem forbids bimetric repulsive gravity. [MH, M. Wohlfarth '09]
- Solution: $N \geq 3$ metrics g_{ab}^I and standard model copies ψ^I .

Action and equations of motion

- N metric tensors and N standard model copies.
- Matter action: sum of standard model actions.
- Gravitational action:

$$S_G[g^1, \dots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \left[\sum_{I,J=1}^N c^{IJ} g^{Iij} R_{ij}^J + F(S^{IJ}) \right].$$

- Symmetric volume form $g_0 = (g^1 g^2 \dots g^N)^{1/N}$.
- $F(S^{IJ})$ quadratic in connection difference tensors $S^{IJ} = \Gamma^I - \Gamma^J$.

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⇒ Equations of motion:

$$T_{ab}^I = \sqrt{\frac{g_0}{g^I}} \left[-\frac{1}{2N} g_{ab}^I \sum_{J,K=1}^N c^{JK} g^{Jij} R_{ij}^K + \sum_{J=1}^N c^{IJ} R_{ab}^J \right]$$

+ terms linear in $\nabla^I S^{JK}$

+ terms quadratic in S^{IJ} .

⇒ Repulsive Newtonian limit for $N \geq 3$. [MH, M. Wohlfarth '10]

Excursion: Area Metric Gravity

- Consider GR + QED, string theory, loop quantum gravity. . .
- ⇒ Area metric naturally appears as quantum geometry.
- Describe spacetime geometry by area measure: [Schuller, Wohlfarth '06]

$$G = G_{abcd}(dx^a \wedge dx^b) \otimes (dx^c \wedge dx^d).$$

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$$G = \sum_{i=1}^N \sigma_{(i)} G_{g^{(i)}}, \quad \sigma_{(i)} = \pm 1.$$

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- Area metric gravity features accelerating expansion. [Punzi, Schuller, Wohlfarth '07]
- **Cosmology of multimetric repulsive gravity?**

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Cosmological symmetry

- Standard cosmology: Robertson–Walker metrics

$$g^I = -n_I^2(t) dt \otimes dt + a_I^2(t) \gamma_{\alpha\beta} dx^\alpha \otimes dx^\beta.$$

- Lapse functions n_I .
- Scale factors a_I .
- Spatial metric $\gamma_{\alpha\beta}$ of constant curvature $k \in \{-1, 0, 1\}$ and Riemann tensor $R(\gamma)_{\alpha\beta\gamma\delta} = 2k\gamma_{\alpha[\gamma}\gamma_{\delta]\beta}$.

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- Perfect fluid matter:

$$T^{Iab} = (\rho^I + p^I) u^{Ia} u^{Ib} + p^I g^{Iab}.$$

- Densities ρ^I .
- Pressures p^I .
- Velocities u^I .
- Normalization: $g_{ab}^I u^{Ia} u^{Ib} = -1$.

Simple cosmological model

- Early universe: radiation; late universe: dust.
- Copernican principle: common evolution for all matter sectors.

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- ⇒ Single effective energy-momentum tensor $T_{ab}^I = T_{ab}$.
- ⇒ Single effective metric $g_{ab}^I = g_{ab}$.
- ⇒ Common scale factors $a^I = a$ and lapse functions $n^I = n$.
- ⇒ Rescale cosmological time to set $n \equiv 1$.
- ⇒ Ricci tensors $R_{ab}^I = R_{ab}$ become equal.
- ⇒ Connection differences $S^{IJi}_{jk} = 0$ vanish.

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- ⇒ Ricci tensors $R_{ab}^I = R_{ab}$ become equal.
- ⇒ Connection differences $S^{IJi}_{jk} = 0$ vanish.
- ⇒ Equations of motion simplify for repulsive Newtonian limit:

$$(2 - N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}.$$

⇒ Negative effective gravitational constant for early / late universe.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$\rho = \frac{3}{2-N} \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right),$$

$$p = -\frac{1}{2-N} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right).$$

⇒ Positive matter density $\rho > 0$ requires $k = -1$ and $\dot{a}^2 < 1$.

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- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{N - 2}{6} (\rho + 3p).$$

⇒ Acceleration must be positive for standard model matter.

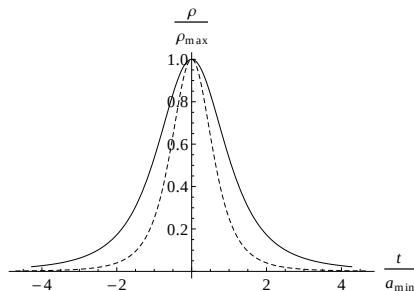
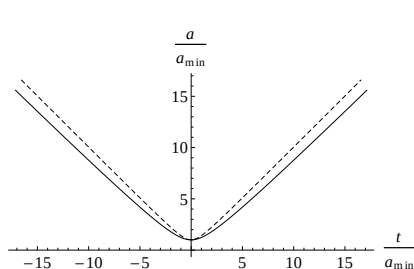
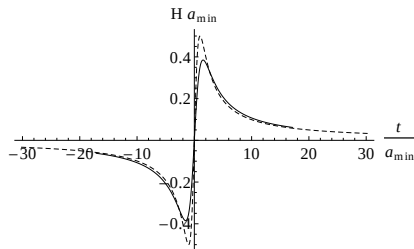
Explicit solution

- Equation of state: $p = \omega \rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time η defined by $dt = a d\eta$:

$$a = a_{\min} \left(\cosh \left(\frac{3\omega + 1}{2} \eta \right) \right)^{\frac{2}{3\omega + 1}},$$
$$\rho = \frac{3}{(N - 2)a_{\min}^2} \left(\cosh \left(\frac{3\omega + 1}{2} \eta \right) \right)^{-\frac{6\omega + 6}{3\omega + 1}}.$$

$\Rightarrow$ Positive minimal radius $a_{\min}$ (Big Bounce). [MH, M. Wohlfarth '10]

Cosmological evolution



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Ingredients

- Metrics $g_{ab}^I = g_{ab}^0 + h_{ab}^I$ with

$$g^0 = -dt \otimes dt + a^2(t) \gamma_{\alpha\beta} dx^\alpha \otimes dx^\beta$$

and $a(t)$ determined by cosmology.

- Scale for structure formation $\ll$ curvature radius of the universe:
 - Cubic volume $0 \leq x^\alpha \leq \ell$.
 - Approximate $\gamma_{\alpha\beta}$ by $\delta_{\alpha\beta}$.
 - Periodic boundary conditions.
- Matter content: n point masses M for each sector.
 - Model for dust matter: $p = 0$.
 - Matter density:

$$\rho = \frac{Mn}{(a\ell)^3}.$$

- Large mean distance $a\ell/\sqrt[3]{Nn} \gg 2GM$.
- Small peculiar velocities $|v_{II}^\alpha| = |a\dot{x}_{II}^\alpha| \ll 1$.

Local dynamics

- Masses of type l follow geodesics of their metric g_{ab}^l :

$$\ddot{x}_{li}^\alpha = \frac{\partial_\alpha h_{00}^l}{2a^2} - 2\frac{\dot{a}}{a}\dot{x}_{li}^\alpha.$$

- Antisymmetric Poisson equation:

$$h_{00}^l = -2 \sum_{J=1}^N (2\delta^{lJ} - 1) \Phi^J.$$

- Individual Newtonian potentials $\Phi^l(t, \vec{x})$:

$$\Phi^l(t, \vec{x}) = -\frac{M}{a(t)} \sum_{i=1}^n \frac{1}{d(\vec{x}, \vec{x}_{li}(t))}.$$

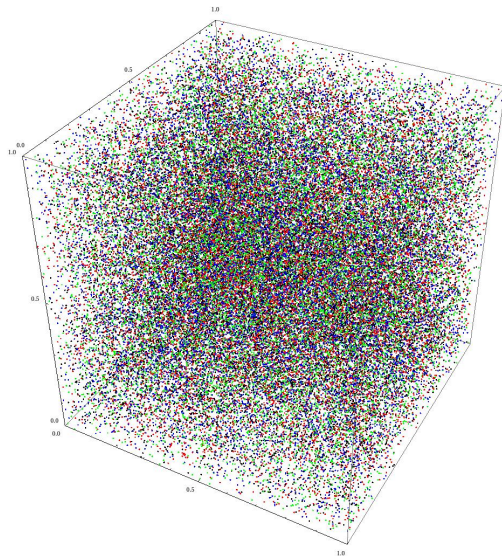
- Periodic distance function $d(\vec{x}, \vec{x}')$:

$$d(\vec{x}, \vec{x}') = \min_{\vec{k} \in \mathbb{Z}^3} \left| \vec{x} - \vec{x}' + \ell \vec{k} \right|.$$

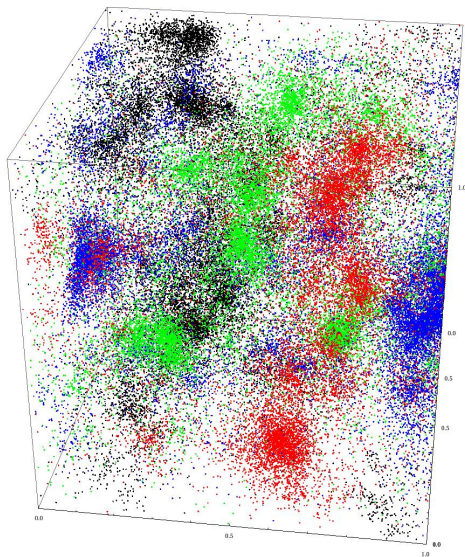
Implementation

- $N = 4$ matter types.
 - $n = 16384$ point masses for each matter type.
 - 2000 calculation steps.
 - Simulation written in C.
 - Calculation using 3.0 GHz Intel Core 2 Duo E8400 CPU.
- ⇒ 7.5 days computation time.

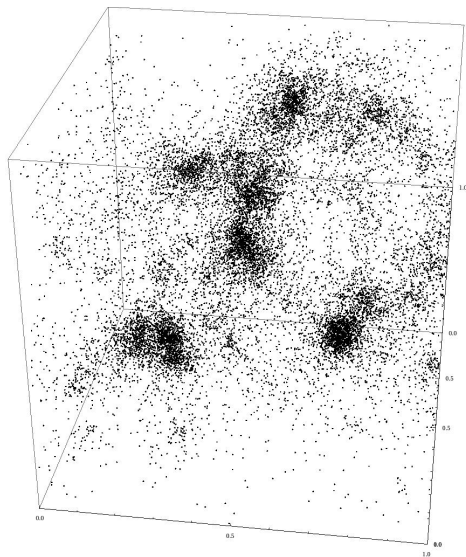
Evolution - all matter types ($N = 4$, $n = 16384$)



Final state - all matter types ($N = 4$, $n = 16384$)



Final state - only visible matter ($N = 4$, $n = 16384$)



Intermediate result

- Different matter types separate.
- Formation of clusters.
- Seemingly empty voids contain invisible matter types.
- Structures are very coarse.

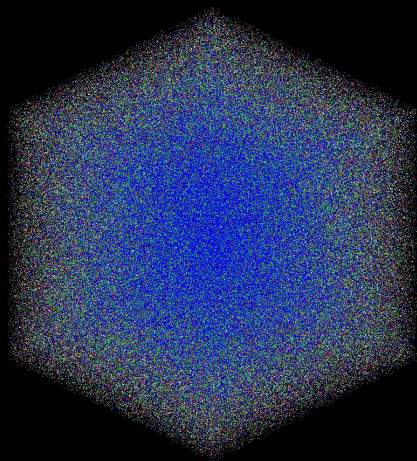
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- ⇒ Increase number of point masses.
- $N = 4$ matter types.
 - $n = 262144$ point masses for each matter type.
 - 17120 calculation steps.

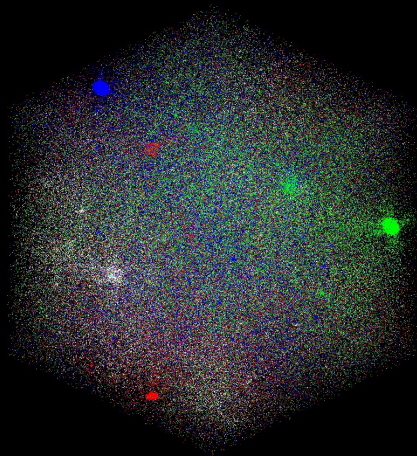
Intermediate result

- Different matter types separate.
 - Formation of clusters.
 - Seemingly empty voids contain invisible matter types.
 - Structures are very coarse.
- ⇒ Increase number of point masses.
- $N = 4$ matter types.
 - $n = 262144$ point masses for each matter type.
 - 17120 calculation steps.
- ⇒ Higher computation power required.
- Simulation written in CUDA.
 - Calculation using NVidia Tesla C2075 GPU.
- ⇒ 2 months computation time.

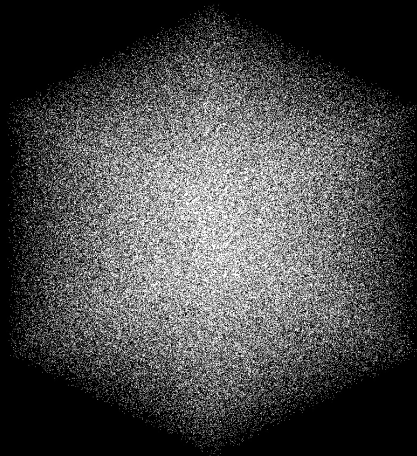
Evolution - all matter types ($N = 4, n = 262144$)



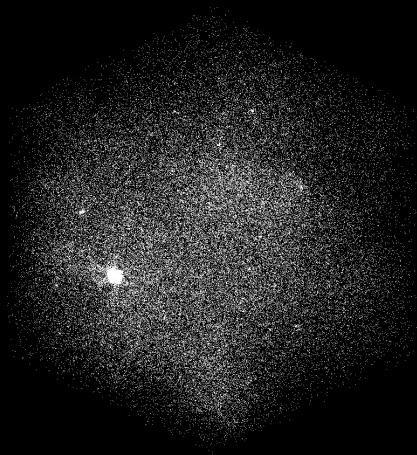
Final state - all matter types ($N = 4$, $n = 262144$)



Evolution - only visible matter ($N = 4$, $n = 262144$)



Final state - only visible matter ($N = 4$, $n = 262144$)



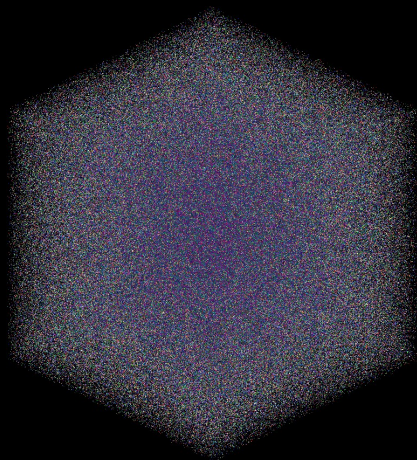
Intermediate result

- Structures are still very coarse.
- Voids are not very empty.
- Violent dynamics.

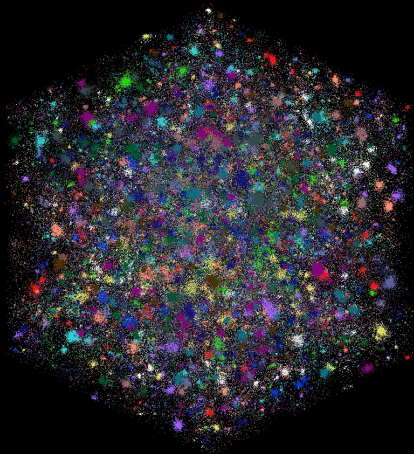
Intermediate result

- Structures are still very coarse.
 - Voids are not very empty.
 - Violent dynamics.
- ⇒ Increase number of matter types.
- $N = 16$ matter types.
 - $n = 65536$ point masses for each matter type.
 - 31600 calculation steps.
 - Simulation written in CUDA.
 - Calculation using NVidia Tesla C2075 GPU.
- ⇒ 3.5 months computation time.

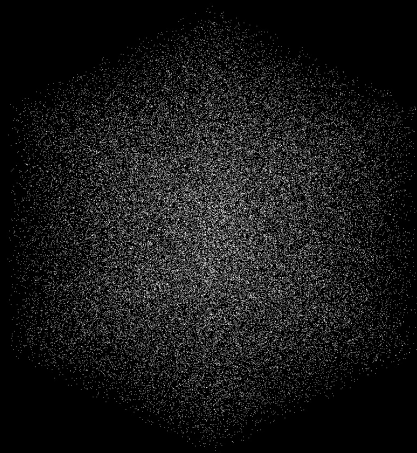
Evolution - all matter types ($N = 16$, $n = 65536$)



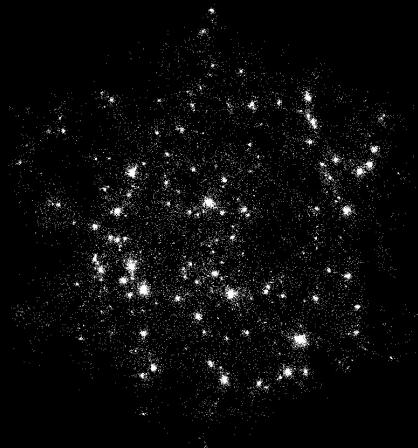
Final state - all matter types ($N = 16$, $n = 65536$)



Evolution - only visible matter ($N = 16, n = 65536$)



Final state - only visible matter ($N = 16, n = 65536$)



- Finer structures.
 - Filaments between galactic clusters.
 - Large voids free of visible matter.
 - Voids contain clusters of repulsively interacting matter.
 - Possible explanation for local velocity anomaly? [Tully '07]
 - Important contribution to weak lensing.
 - Strong negative gravitational lenses?
- ⇒ Calculate gravitational lensing from simulation data!

Results

- Finer structures.
- Filaments between galactic clusters.
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 - Possible explanation for local velocity anomaly? [Tully '07]
 - Important contribution to weak lensing.
 - Strong negative gravitational lenses?
- ⇒ Calculate gravitational lensing from simulation data!
- Further increase number of point masses used in the simulation?
- Problem: Currently used algorithm scales as $\mathcal{O}(n^2)$.

⇒ Use different algorithm!

- Adapt GADGED-2 code [Springel '05] to multimetric gravity.
- TreeSPH algorithm:
 - Gravitational forces from hierarchical multipole expansion.
 - Gas dynamics from smoothed particle hydrodynamics (SPH).
- Better scaling behavior $\mathcal{O}(n \log n)$.
- Usable on multicore PCs & clusters.

Outline

- 1 Introduction
- 2 Multimetric cosmology
- 3 Simulation of structure formation
- 4 Conclusion**

- Idea: Repulsive gravity might explain dark matter & dark energy.
- ⇒ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.

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- ⇒ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.
- Cosmology:
 - ⇒ Big Bounce cosmology.
 - ⇒ Accelerating expansion becomes small at late times.
- Structure formation:
 - ⇒ Result highly depends on number N of matter types.
 - ⇒ Formation of galactic clusters.
 - ⇒ Voids contain repulsively interacting, invisible matter.

- Further cosmological calculations:
 - Analyze stability of cosmological solutions.
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- Improved simulations of structure formation:
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- Connection to observations:
 - Test multimetric cosmology against CMB fluctuations.
 - Peculiar motion of galaxies due to repulsive matter?
 - Repulsive matter distribution in voids from weak lensing?
 - Search for negative gravitational lenses.