Eksperimentaalne kokkusobivus multimeetrilise gravitatsiooni teooriaga

Parametriseeritud post-Newton'i formalismi laiendus $N \geq 2$ meetrilistele tensoritele

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Experimental consistency of multimetric gravity

An extension of the PPN formalism to $N \geq 2$ metrics

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Outline

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Einstein gravity

- Gravity is described by metric tensor g_{ab}.
- Einstein-Hilbert action:

$$S_G = \frac{1}{16\pi} \int \omega R \,.$$

  - Volume form ω.
  - Scalar curvature R.
- Minimally coupled matter action:

$$S_M = \int \omega \mathcal{L}_M \,.$$

- Einstein equations:

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab} \,.$$

Application to the universe

- 4.9% visible matter. [*PLANCK coll.* '13]

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- 26.8% dark matter.
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  - Anomalous light deflection. [Wambsganss '98]
  - Formation of large-scale structures [Peebles '80]

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$\Rightarrow$ Problem: What are dark matter and dark energy?

Explanations for the dark universe

- Particle physics:
  - Dark matter: [Bertone, Hooper, Silk '05]
    - Weakly interacting massive particles (WIMPs). [Ellis *et al.* '84]
    - Axions. [Preskill, Wise, Wilczek '83]
    - Massive compact halo objects (MACHOs). [Paczynski '86]
  - Dark energy: [Copeland, Sami, Tsujikawa '06]
    - Quintessence. [Peebles, Ratra '88]
    - K-essense. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
    - Chaplygin gas. [Kamenshchik, Moschella, Pasquier '01]
- Gravity:
  - Modified Newtonian dynamics (MOND). [Milgrom '83]
  - Tensor-vector-scalar theories. [Bekenstein '04]
  - Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
  - Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
  - Non-symmetric gravity. [Moffat '95]
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  - New idea: repulsive gravity $\Leftrightarrow$ negative mass!

Repulsive gravity effects

- Idea here: Additional standard model copy.
- Only interaction between both copies: repulsive gravity.
  - $\Rightarrow$ Each type of matter appears dark to the other one.
  - $\Rightarrow$ Both types of matter repel each other.

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  - $\Rightarrow$ Both types of matter repel each other.
- Universe contains equal amounts of both types of matter:
  - $\rightsquigarrow$ Dark galaxies “push” visible matter & light towards visible galaxies.
  - $\Rightarrow$ Explanation of dark matter!
  - $\rightsquigarrow$ Mutual repulsion between galaxies drives accelerating expansion.
  - $\Rightarrow$ Explanation of dark energy!

Repulsive Einstein gravity

- Two standard model copies $\varphi^{\pm}$ for positive and negative mass.
- Two different types of test masses follow different trajectories.
- Two types of test mass trajectories $\Rightarrow$ two types of observers.
- Observer trajectories are autoparallels of two connections $\nabla^{\pm}$.
- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.

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- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.
- No-go theorem forbids bimetric repulsive gravity. [MH, M. Wohlfarth '09]

$\Rightarrow$ Solution: $N \geq 3$ metrics g^{I}_{ab} and standard model copies φ^{I}.

$\Rightarrow$ Multimetric gravity.

Multimetric gravity

- Consider multimetric gravity with...
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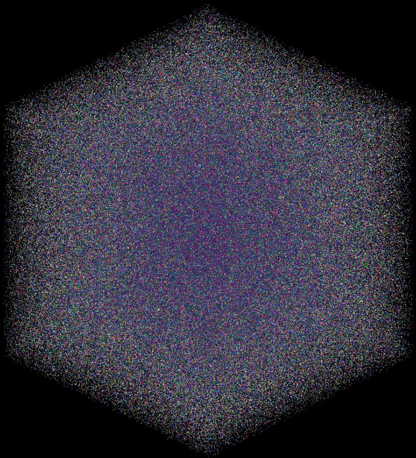
$\Rightarrow$ Compatible with PPN bounds at linearized level. [MH, M. Wohlfarth '10]

$\Rightarrow$ Testable using gravitational waves. [MH '11]

$\Rightarrow$ Structure formation features clusters and voids.

Structure formation - all matter types



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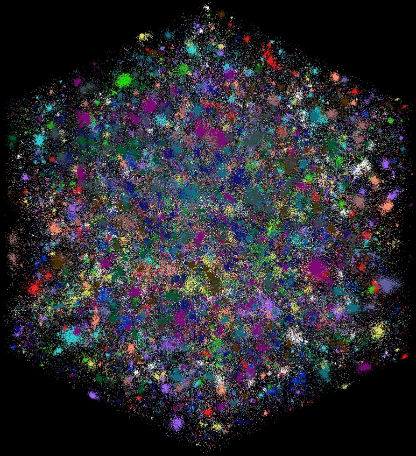
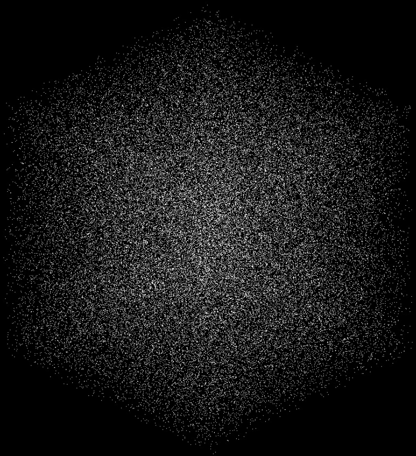


Structure formation - only visible matter



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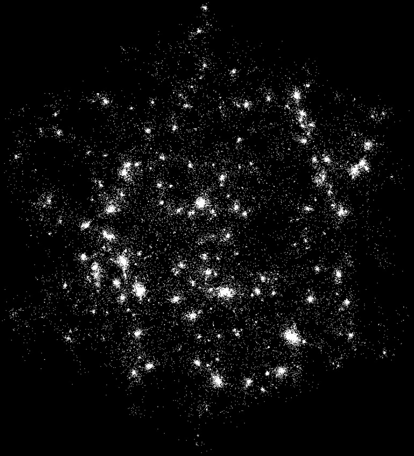

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- Experimental consistency at full post-Newtonian level?

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Assumptions

- Gravitational field equations:

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- Slow-moving source matter.

$\Rightarrow$ Expand quantities in “velocity” orders $\mathcal{O}(n)$.

- Weak gravitational field.

$\Rightarrow$ Expand metric around flat background:

$$g^I_{ab} = \eta_{ab} + h^I_{ab} = \eta_{ab} + h^{I(1)}_{ab} + h^{I(2)}_{ab} + h^{I(3)}_{ab} + h^{I(4)}_{ab} + \mathcal{O}(5)\,.$$

- Each term $h^{I(n)}_{ab}$ is of order $\mathcal{O}(n)$.

Post-Newtonian metric

- Post-Newtonian metric ansatz:

$$h_{00}^{I(2)} = -\sum_{J=1}^{N} \alpha^{IJ} \triangle \chi^J \,,$$

$$h_{\alpha\beta}^{I(2)} = \sum_{J=1}^{N} \left(2\theta^{IJ} \chi^J{}_{,\alpha\beta} - (\gamma^{IJ} + \theta^{IJ}) \triangle \chi^J \delta_{\alpha\beta}\right) \,,$$

$$h_{0\alpha}^{I(3)} = \sum_{J=1}^{N} \left(\sigma_+^{IJ} W_\alpha^{J+} + \sigma_-^{IJ} W_\alpha^{J-}\right) \,,$$

$$h_{00}^{I(4)} = \sum_{J=1}^{N} \left(\phi_p^{IJ} \Phi_p^J + \phi_\Pi^{IJ} \Phi_\Pi^J + \sum_{A=1}^{2} \omega_A^{IJ} \Omega_A^J\right) + \sum_{J,K=1}^{N} \sum_{A=1}^{7} \psi_A^{IJK} \Psi_A^{JK} \,.$$

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- Parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_\pm^{IJ}, \phi_p^{IJ}, \phi_\Pi^{IJ}, \omega_1^{IJ}, \omega_2^{IJ}, \psi_1^{IJK}, \ldots, \psi_7^{IJK}$.

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- Potentials $\chi^{I}, W^{I\pm}, \Phi_{p}^{I}, \Phi_{\Pi}^{I}, \Omega_{1}^{I}, \Omega_{2}^{I}, \Psi_{1}^{IJ}, \ldots, \Psi_{7}^{IJ}$.

PPN potentials - part 1

- Superpotential:

$$\chi^I = -\int \rho'^I |\vec{x} - \vec{x}'| d^3x' \,.$$

- Vector potentials:

$$W_\alpha^{\pm I} = \int \rho'^I \left(\frac{v'^I_\alpha}{|\vec{x} - \vec{x}'|} \pm \frac{(x_\alpha - x'_\alpha)(x_\beta - x'_\beta) v'^I_\beta}{|\vec{x} - \vec{x}'|^3} \right) d^3x' \,.$$

- Pressure:

$$\Phi_p^I = \int \frac{p'^I}{|\vec{x} - \vec{x}'|} d^3x' \,.$$

- Internal energy:

$$\Phi_\Pi^I = \int \frac{\rho'^I \Pi'^I}{|\vec{x} - \vec{x}'|} d^3x' \,.$$

PPN potentials - part 2

- Kinetic energy:

$$\Omega_1^I = \int \frac{\rho'^I v'^{I2}}{|\vec{x}-\vec{x}'|} d^3x' \,, \quad \Omega_2^I = \int \frac{\rho'^I \left[\vec{v}'^I \cdot (\vec{x}-\vec{x}')\right]^2}{|\vec{x}-\vec{x}'|^3} d^3x' \,.$$

- Non-linear potentials:

$$\begin{aligned}
\triangle\triangle\Psi_1^{IJ} &= \triangle\chi^I\triangle\triangle\triangle\chi^J \,, & \triangle\triangle\Psi_2^{IJ} &= \chi^I_{,\alpha\beta}\triangle\triangle\chi^J_{,\alpha\beta} \,,\\
\triangle\triangle\Psi_3^{IJ} &= \triangle\chi^I_{,\alpha}\triangle\triangle\chi^J_{,\alpha} \,, & \triangle\triangle\Psi_4^{IJ} &= \chi^I_{,\alpha\beta\gamma}\triangle\chi^J_{,\alpha\beta\gamma} \,,\\
\triangle\triangle\Psi_5^{IJ} &= \triangle\triangle\chi^I\triangle\triangle\chi^J \,, & \triangle\triangle\Psi_6^{IJ} &= \triangle\chi^I_{,\alpha\beta}\triangle\chi^J_{,\alpha\beta} \,,
\end{aligned}$$

$$\triangle\triangle\Psi_7^{IJ} = \chi^I_{,\alpha\beta\gamma\delta}\chi^J_{,\alpha\beta\gamma\delta} \,.$$

$\rightsquigarrow$ Non-linearity in superposition law.
$\rightsquigarrow$ Gravitational self-energy.
$\rightsquigarrow$ ...

Matter content

- Perfect fluid energy-momentum tensor:

$$T^I_{00} = \rho^I\left(1 + \Pi^I + {v^I}^2 + \sum_{J=1}^{N}\alpha^{IJ}\triangle\chi^J\right) + \mathcal{O}(6)\,,$$
$$T^I_{0\alpha} = -\rho^I v^I_\alpha + \mathcal{O}(5)\,,$$
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\end{aligned}
$$

- Covariant energy-momentum conservation $\nabla^I_a T^{I\,ab} = 0$:
  - Continuity equation:

$$
0 = \nabla^I_a T^{I\,a0} = \rho^I_{,0} + (\rho^I v^I_\alpha)_{,\alpha} + \mathcal{O}(5)\,.
$$

  - Eulerian equation of motion:

$$
0 = \nabla^I_a T^{I\,a\alpha} = \rho^I\frac{dv^I_\alpha}{dt} + p^I_{,\alpha} + \frac{1}{2}\rho^I\sum_{J=1}^{N}\alpha^{IJ}\triangle\chi^J_{,\alpha} + \mathcal{O}(6)\,.
$$

Gauge transformations

- Invariance of the action under diffeomorphisms.
- Diffeomorphism generated by vector field ξ.
- Tensor fields change by their Lie derivatives:

$$\delta_\xi g^I_{ab} = (\mathcal{L}_\xi g^I)_{ab} = 2\nabla^I_{(a}\xi_{b)}\,.$$

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- Require form-invariance of the PPN metric.

$\Rightarrow$ Vector field must take the form

$$\xi_0 = \sum_{I=1}^{N} \lambda^I_1 \chi^I_{,0}\,, \quad \xi_\alpha = \sum_{I=1}^{N} \lambda^I_2 \chi^I_{,\alpha}\,.$$

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- Use gauge invariance to eliminate potentials from the PPN metric.

$\Rightarrow$ PPN parameters $\theta^{II} = 0$ and $\psi^{III}_1 = \psi^{III}_5$ in standard gauge.

Lorentz transformations

- Transform metric to moving coordinate system.
- Relative velocity $\vec{w}$ of order $\mathcal{O}(1)$.
- Express PPN potentials in new coordinate system.

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$\Rightarrow$ New $\vec{w}$ dependent terms in the PPN metric appear.

$\Rightarrow$ New terms vanish if and only if

$$\begin{aligned}
\alpha^{IJ} + \gamma^{IJ} + \theta^{IJ} + \sigma_{+}^{IJ} &= 0\,,\\
2\sigma_{+}^{IJ} + \omega_{1}^{IJ} + \omega_{2}^{IJ} &= 0\,,\\
\alpha^{IJ} + 2\theta^{IJ} - 2\sigma_{-}^{IJ} - \omega_{1}^{IJ} &= 0\,,\\
2\theta^{IJ} + \sigma_{+}^{IJ} - \sigma_{-}^{IJ} - 2\theta^{II} - \sigma_{+}^{II} + \sigma_{-}^{II} &= 0\,.
\end{aligned}$$

$\Rightarrow$ Simple test for Lorentz invariance of a gravity theory.

Order-wise solution of field equations

- Second velocity order $\mathcal{O}(2)$:
  - Solve field equations $K_{00}^{I(2)} = 8\pi T_{00}^{I(2)}$ and $K_{\alpha\beta}^{I(2)} = 8\pi T_{\alpha\beta}^{I(2)}$.
  - Determine metric components $h_{00}^{I(2)}$ and $h_{\alpha\beta}^{I(2)}$.

$\Rightarrow$ Obtain PPN parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}$.

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- Third velocity order $\mathcal{O}(3)$:
  - Solve field equations $K^{I(3)}_{0\alpha} = 8\pi T^{I(3)}_{0\alpha}$.
  - Determine metric components $h^{I(3)}_{0\alpha}$.

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- Fourth velocity order $\mathcal{O}(4)$:
  - Solve field equations $K^{I(4)}_{00} = 8\pi T^{I(4)}_{00}$ and $K^{I(4)}_{\alpha\beta} = 8\pi T^{I(4)}_{\alpha\beta}$.
  - Determine metric component $h^{I(4)}_{00}$.

  $\Rightarrow$ Obtain PPN parameters $\sigma^{IJ}_{-}, \phi^{IJ}_{p}, \phi^{IJ}_{\Pi}, \omega^{IJ}_{1}, \omega^{IJ}_{2}, \psi^{IJK}_{1}, \ldots, \psi^{IJK}_{7}$.
  - σ^{IJ}_{-} determined through gauge fixing.

Outline

Standard PPN formalism

- Only one metric g_{ab} and corresponding matter source T_{ab}.
- Standard PPN metric:

$$
\begin{aligned}
h^{(2)}_{00} &= 2\alpha U\,, \\
h^{(2)}_{\alpha\beta} &= 2\gamma U\delta_{\alpha\beta}\,, \\
h^{(3)}_{0\alpha} &= -\frac{1}{2}(3+4\gamma+\alpha_1-\alpha_2+\zeta_1-2\xi)V_\alpha \\
&\quad -\frac{1}{2}(1+\alpha_2-\zeta_1+2\xi)W_\alpha\,, \\
h^{(4)}_{00} &= -2\beta U^2 - 2\xi\Phi_W + (2+2\gamma+\alpha_3+\zeta_1-2\xi)\Phi_1 \\
&\quad +2(1+3\gamma-2\beta+\zeta_2+\xi)\Phi_2 + 2(1+\zeta_3)\Phi_3 \\
&\quad +2(3\gamma+3\zeta_4-2\xi)\Phi_4 - (\zeta_1-2\xi)\mathcal{A}\,.
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&\quad - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_\alpha\,, \\
h^{(4)}_{00} &= -2\beta U^2 - 2\xi\Phi_W + (2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 \\
&\quad + 2(1 + 3\gamma - 2\beta + \zeta_2 + \xi)\Phi_2 + 2(1 + \zeta_3)\Phi_3 \\
&\quad + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A}\,.
\end{aligned}
$$

- PPN parameters $\alpha, \gamma, \beta, \alpha_1, \ldots, \alpha_3, \zeta_1, \ldots, \zeta_4, \xi$.

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&\quad + 2(3\gamma+3\zeta_4-2\xi)\Phi_4 - (\zeta_1-2\xi)\mathcal{A}\,.
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$$

- PPN parameters $\alpha, \gamma, \beta, \alpha_1, \ldots, \alpha_3, \zeta_1, \ldots, \zeta_4, \xi$.
- PPN potentials U, V_α, W_α, $\Phi_1, \ldots, \Phi_4, \Phi_W, \mathcal{A}$.

Standard PPN parameters

Parameter	Value	Interpretation / effect
α	1	Gravitational constant
γ	1	Light deflection
β	1	Perihelion precession
α_1	0	Orbit polarization
α_2	0	Spin precession
α_3	0	Self-acceleration
ζ_1	0	Combined effects, e.g., Nordtvedt effect
ζ_2	0	Binary pulsar acceleration
ζ_3	0	Newton's third law
ζ_4	0	Active / passive gravitational mass ratio
ξ	0	Earth tides

Translation of PPN potentials

- Identify visible matter $T_{ab} \equiv T^1_{ab}$.
- Standard in terms of multimetric PPN potentials:

$$U = -\frac{1}{2}\triangle\chi^1\,, \quad U^2 = \frac{1}{2}\Psi^{11}_1 + 2\Psi^{11}_3 + \frac{1}{2}\Psi^{11}_5 + \Psi^{11}_6\,,$$

$$V_\alpha = \frac{W^{+1}_\alpha + W^{-1}_\alpha}{2}\,, \quad W_\alpha = \frac{W^{+1}_\alpha - W^{-1}_\alpha}{2}\,,$$

$$\Phi_1 = \Omega^1_1\,, \quad \Phi_2 = \frac{1}{4}\Psi^{11}_1 + \frac{1}{2}\Psi^{11}_3 + \frac{1}{4}\Psi^{11}_5\,, \quad \Phi_3 = \Phi^1_\Pi\,, \quad \Phi_4 = \Phi^1_p\,,$$

$$\Phi_W = -\frac{1}{4}\Psi^{11}_1 - \Psi^{11}_2 - \frac{5}{2}\Psi^{11}_3 - 2\Psi^{11}_4 - \frac{1}{4}\Psi^{11}_5 - 3\Psi^{11}_6\,, \quad \mathcal{A} = \Omega^1_2\,.$$

- Larger number of multimetric vs. standard potentials.

$\Rightarrow$ Cannot express all multimetric in terms of standard potentials.

$\Rightarrow$ Multimetric PPN formalism is more general.

Translation of PPN parameters

- Identify metric $g_{ab} \equiv g^1_{ab}$ detectable using visible matter.
- Multimetric in terms of standard PPN parameters:

$$
\begin{gathered}
\alpha^{11} = \alpha\,, \quad \sigma_-^{11} = -\frac{1}{2} - \gamma - \frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \frac{1}{2}\zeta_1 + \xi\,, \\
\gamma^{11} = \gamma\,, \quad \sigma_+^{11} = -1 - \gamma - \frac{1}{4}\alpha_1\,, \quad \phi_\Pi^{11} = 2 + 2\zeta_3\,, \\
\phi_p^{11} = 6\gamma + 6\zeta_4 + 4\xi\,, \quad \omega_1^{11} = 2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi\,, \\
\omega_2^{11} = 2\xi - \zeta_1\,, \quad \psi_2^{111} = 2\xi\,, \quad \psi_6^{111} = 6\xi - 2\beta\,, \\
\psi_1^{111} = \psi_5^{111} = \frac{1}{2} + \frac{3}{2}\gamma - 2\beta + \frac{1}{2}\zeta_2 + \xi\,, \quad \psi_4^{111} = 4\xi\,, \\
\psi_3^{111} = 1 + 3\gamma - 6\beta + \zeta_2 + 6\xi\,, \quad \theta^{11} = \psi_7^{111} = 0\,.
\end{gathered}
$$

Translation of PPN parameters

- Identify metric $g_{ab} \equiv g^1_{ab}$ detectable using visible matter.
- Multimetric in terms of measured standard PPN parameters:

$$\alpha^{11} = \alpha = 1\,, \quad \sigma^{11}_{-} = -\frac{1}{2} - \gamma - \frac{1}{4}\alpha_1 + \frac{1}{2}\alpha_2 - \frac{1}{2}\zeta_1 + \xi = -\frac{3}{2}\,,$$

$$\gamma^{11} = \gamma = 1\,, \quad \sigma^{11}_{+} = -1 - \gamma - \frac{1}{4}\alpha_1 = -2\,, \quad \phi^{11}_{\Pi} = 2 + 2\zeta_3 = 2\,,$$

$$\phi^{11}_{p} = 6\gamma + 6\zeta_4 + 4\xi = 6\,, \quad \omega^{11}_{1} = 2 + 2\gamma + \alpha_3 + \zeta_1 - 2\xi = 4\,,$$

$$\omega^{11}_{2} = 2\xi - \zeta_1 = 0\,, \quad \psi^{111}_{2} = 2\xi = 0\,, \quad \psi^{111}_{6} = 6\xi - 2\beta = -2\,,$$

$$\psi^{111}_{1} = \psi^{111}_{5} = \frac{1}{2} + \frac{3}{2}\gamma - 2\beta + \frac{1}{2}\zeta_2 + \xi = 0\,, \quad \psi^{111}_{4} = 4\xi = 0\,,$$

$$\psi^{111}_{3} = 1 + 3\gamma - 6\beta + \zeta_2 + 6\xi = -2\,, \quad \theta^{11} = \psi^{111}_{7} = 0\,.$$

$\Rightarrow$ $\theta^{11} = 0$ and $\psi^{111}_1 = \psi^{111}_5$ due to gauge choice.

$\Rightarrow$ $\alpha^{11} = 1$ can be achieved by choice of units.

$\Rightarrow$ 13 physical parameters accessible to visible matter experiments.

Outline

Action - part 1

Generic, simple, multimetric gravity action:

$$
\begin{aligned}
S_G = {} & \frac{1}{16\pi}\int d^4x \\
& \sqrt{g_0}\Bigg[\sum_{I=1}^{N}\Big(c_1 R^I + g^{I\,ij}\left(c_3\tilde{S}^I{}_i\tilde{S}^I{}_j + c_5\tilde{S}^I{}_k\tilde{S}^{I\,k}{}_{ij} + c_7\tilde{S}^{I\,k}{}_{il}\tilde{S}^{I\,l}{}_{jk}\right) \\
& + g^{I\,ij}g^{I\,kl}g^I{}_{mn}\left(c_9\tilde{S}^{I\,m}{}_{ik}\tilde{S}^{I\,n}{}_{jl} + c_{11}\tilde{S}^{I\,m}{}_{ij}\tilde{S}^{I\,n}{}_{kl}\right)\Big) \\
& + \sum_{I,J=1}^{N}\Big(c_2 g^{I\,ij}R^J{}_{ij} + g^{I\,ij}\left(c_4 S^{IJ}{}_i S^{IJ}{}_j + c_6 S^{IJ}{}_k S^{IJ\,k}{}_{ij} + c_8 S^{IJ\,k}{}_{il}S^{IJ\,l}{}_{jk}\right) \\
& + g^{I\,ij}g^{I\,kl}g^I{}_{mn}\left(c_{10}S^{IJ\,m}{}_{ik}S^{IJ\,n}{}_{jl} + c_{12}S^{IJ\,m}{}_{ij}S^{IJ\,n}{}_{kl}\right)\Big)\Bigg]\,.
\end{aligned}
$$

Action - part 2

- Ricci tensor and Ricci scalar:

$$R^I\,, \qquad g^{I\,ab}R^J{}_{ab}\,.$$

- Connection difference tensors:

$$S^{IJ\,i}{}_{jk} = \Gamma^{I\,i}{}_{jk} - \Gamma^{J\,i}{}_{jk}\,, \qquad S^{IJ}{}_j = S^{IJ\,k}{}_{jk}\,,$$

$$\tilde{S}^{J\,i}{}_{jk} = \frac{1}{N}\sum_{I=1}^{N} S^{IJ\,i}{}_{jk}\,, \qquad \tilde{S}^J{}_j = \tilde{S}^{J\,k}{}_{jk}\,.$$

- Mixed volume form

$$g_0 = \prod_{I=1}^{N}\left(g^I\right)^{\frac{1}{N}}\,.$$

- 12 free, constant parameters:

$$c_1, \ldots, c_{12}\,.$$

Permutation symmetry

- Consider permutation $I \mapsto \pi(I) = \tilde{I}$ of sectors.
- Action is symmetric under arbitrary permutations:

$$S[g^I, \varphi^I] = S[g^{\tilde{I}}, \varphi^{\tilde{I}}] \,.$$

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- All constant expansion coefficients $P^{I_1 \cdots I_n}$ are symmetric:

$$P^{I_1 \cdots I_n} = P^{\tilde{I}_1 \cdots \tilde{I}_n}\,.$$

- Most general form for 2 or 3 indices:

$$P_2^{IJ} = \frac{\bar{P}_2}{N} + \hat{P}_2 \delta^{IJ}\,,$$

$$P_3^{IJK} = \frac{\bar{P}_3}{N^2} + \frac{\overleftarrow{P}_3 \delta^{IJ} + \vec{P}_3 \delta^{IK} + \tilde{P}_3 \delta^{JK}}{N} + \hat{P}_3 \delta^{IJ} \delta^{IK}\,.$$

Repulsive Newtonian limit

- All diagonal elements α^{II} are equal.
- Units can be chosen to rescale $\alpha^{II} = 1$.
- All off-diagonal elements $\alpha^{IJ} = z$ are equal.
- Parameter values:

$$\bar{\alpha} = Nz\,, \qquad \hat{\alpha} = 1 - z\,.$$

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- Newtonian metric perturbation:

$$\begin{aligned} h_{00}^{I(2)} &= -\triangle\chi^I - z\sum_{J\neq I}\triangle\chi^J \\ &= 2U^I + 2z\sum_{J\neq I}U^J\,. \end{aligned}$$

$\Rightarrow$ Repulsive Newtonian limit for $z = -1$.

Conditions

- Gauge fixing conditions for standard PPN gauge:

$$\frac{\bar{\theta}}{N} + \hat{\theta} = 0\,,$$

$$\frac{\bar{\psi}_1}{N^2} + \frac{\overleftarrow{\psi}_1 + \vec{\psi}_1 + \tilde{\psi}_1}{N} + \hat{\psi}_1 = \frac{\bar{\psi}_5}{N^2} + \frac{\overleftarrow{\psi}_5 + \vec{\psi}_5 + \tilde{\psi}_5}{N} + \hat{\psi}_5\,.$$

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$\Rightarrow$ Provide independent equations for calculating PPN parameters.

- Experimental consistency with solar system:

$$\frac{\bar{\gamma}}{N} + \hat{\gamma} = 1\,,$$

$$\frac{\bar{\psi}_1}{N^2} + \frac{\overleftarrow{\psi}_1 + \overrightarrow{\psi}_1 + \tilde{\psi}_1}{N} + \hat{\psi}_1 = 0\,,$$

  + similar constraints from other PPN parameters.

$\Rightarrow$ Impose restrictions on viable input parameters $c_1, \ldots, c_{12}$.

Results

- 6 constants $c_4, c_6, c_8, c_{10}, c_{11}, c_{12}$ remain free parameters.
- 6 constants $c_1, c_2, c_3, c_5, c_7, c_9$ fixed depending on N and z.
- Multimetric gravity compatible with experiments. [MH '13]

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- Multimetric gravity compatible with experiments. [MH '13]
- PPN parameters (+ lengthy expressions for $\psi_1^{IJK}, \ldots, \psi_7^{IJK}$):

$$
\begin{aligned}
\bar{\alpha} &= Nz\,, & \hat{\alpha} &= 1 - z\,,\\
\bar{\gamma} &= Nz\,, & \hat{\gamma} &= 1 - z\,,\\
\bar{\theta} &= 0\,, & \hat{\theta} &= 0\,,\\
\bar{\sigma}_+ &= -2Nz\,, & \hat{\sigma}_+ &= 2(z-1)\,,\\
\bar{\sigma}_- &= \frac{N}{2}(1-4z)\,, & \hat{\sigma}_- &= 2(z-1)\,,\\
\bar{\omega}_1 &= N(5z-1)\,, & \hat{\omega}_1 &= 5(1-z)\,,\\
\bar{\omega}_2 &= N(1-z)\,, & \hat{\omega}_2 &= z-1\,,\\
\bar{\phi}_p &= 2N(4z-1)\,, & \hat{\phi}_p &= 8(1-z)\,,\\
\bar{\phi}_\Pi &= 2Nz\,, & \hat{\phi}_\Pi &= 2(1-z)\,.
\end{aligned}
$$

Interpretation for $z = -1$

- Newtonian limit:
  - Diagonal elements: $\alpha^{II} = 1$.

  $\Rightarrow$ Attractive gravity within each matter sector.
  - Off-diagonal elements: $\alpha^{IJ} = -1$ for $I \neq J$.

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  $\Rightarrow$ Light bending away from gravitational matter source.
- Frame dragging / Lense-Thirring effect:
  - Diagonal elements: $\sigma_+^{II} = -2$.

  $\Rightarrow$ Frame dragging follows direction of rotation.
  - Off-diagonal elements: $\sigma_+^{IJ} = 2$ for $I \neq J$.

  $\Rightarrow$ Frame dragging against direction of rotation.

Outline

Cosmological symmetry

- Standard cosmology: Robertson–Walker metrics

$$g^I = -(n^I)^2(t)dt \otimes dt + (a^I)^2(t)\gamma_{\alpha\beta}dx^\alpha \otimes dx^\beta\,.$$

  - Lapse functions n^I.
  - Scale factors a^I.
  - Spatial metric $\gamma_{\alpha\beta}$ of constant curvature $k \in \{-1, 0, 1\}$ and Riemann tensor $R(\gamma)_{\alpha\beta\gamma\delta} = 2k\gamma_{\alpha[\gamma}\gamma_{\delta]\beta}$.

- Normalization: $g^I_{ab}u^{Ia}u^{Ib} = -1$.

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- Perfect fluid matter:

$$T^{I\,ab} = (\rho^I + p^I)u^{Ia}u^{Ib} + p^I g^{Iab} \,.$$

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Simple cosmological model

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  - $\Rightarrow$ All metrics are equal: $g^I_{ab} = g_{ab}$.
  - $\Rightarrow$ Common scale factors $a^I = a$ and lapse functions $n^I = n$.
  - $\Rightarrow$ Rescale cosmological time to set $n \equiv 1$.

$\Rightarrow$ Equations of motion simplify:

$$(c_1 + c_2)\left(R_{ab} - \frac{1}{2}Rg_{ab}\right) = 8\pi T_{ab}\,.$$

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- PPN constraints on c_1 and c_2:

$$c_1 + c_2 = \frac{1}{1 + (N-1)z}\,.$$

$\Rightarrow$ Negative effective gravitational constant for $z = -1$ and $N > 2$.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$8\pi\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),$$

$$8\pi p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right).$$

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- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{4\pi(N-2)}{3}(\rho + 3p).$$

$\Rightarrow$ Acceleration must be positive for standard model matter.

Explicit solution

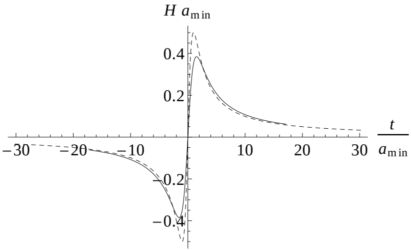
- Equation of state: $p = w\rho$; dust: $w = 0$, radiation: $w = 1/3$.
- General solution using conformal time η defined by $dt = a\,d\eta$:

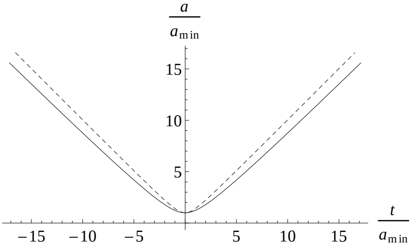
$$a = a_{\text{min}}\left(\cosh\left(\frac{3w+1}{2}(\eta-\eta_0)\right)\right)^{\frac{2}{3w+1}}\,,$$

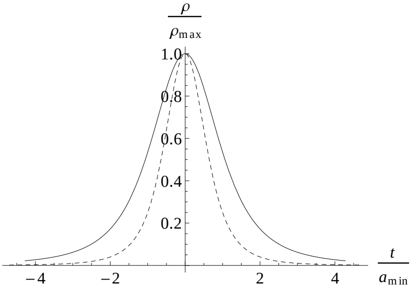
$$\rho = \rho_{\text{max}}\left(\cosh\left(\frac{3w+1}{2}(\eta-\eta_0)\right)\right)^{-\frac{6w+6}{3w+1}}\,.$$

$\Rightarrow$ Positive minimal radius a_{min}. [MH, M. Wohlfarth '10]

Cosmological evolution







Outline

Summary

- Multimetric PPN formalism:
  - Assume perfect fluid matter.
  - Determine post-Newtonian metric.
  - PPN parameters $\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma_{\pm}^{IJ}, \phi_{p}^{IJ}, \phi_{\Pi}^{IJ}, \omega_1^{IJ}, \omega_2^{IJ}, \psi_1^{IJK}, \ldots, \psi_7^{IJK}$.
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  - Extension of standard PPN formalism to $N \geq 2$ metrics.

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  - 13 parameters accessible to visible matter experiments.
  - Extension of standard PPN formalism to $N \geq 2$ metrics.
- Application to multimetric repulsive gravity:
  - Simple model dependent on 12 constant parameters.
  - 6 parameters fixed by experimental consistency.
  - Experimentally consistent model with 6 free parameters.
  - Relation between repulsive gravity and accelerating expansion.

Outlook

- Experimental significance of new visible PPN parameters:
  - Effects of new PPN potentials on current experiments?
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- Post-Newtonian conservation laws:
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- Further experimental tests of multimetric gravity:
  - Strong fields and pulsars: parameterized post-Keplerian formalism?
  - Gravitational waves: parameterized post-Einsteinian formalism?
  - Cosmology: Cosmic microwave background?

Possible Bachelor's theses

- Stability of cosmological solutions
- Improved simulations of structure formation using GADGET2
- Your idea here: ________________

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