Post-Newtonian formalism for multimetric gravity

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Motivation

- Only 5% of the universe are visible.
- 95% are completely unknown.
- Idea: Add negative mass standard model.
- ⇒ Both copies appear mutually dark.
- ⇒ Well-known masses and couplings.
- Galaxies of both types should exist.
- ⇝ Repulsion drives accelerating expansion.

Repulsive Einstein gravity

- Positive and negative test masses.
- ⇒ Two types of observer trajectories.
- ⇒ Autoparallels of two connections $\nabla^{\pm}$.
- ⇒ Attach parallely transported frames.
- ⇒ Orthonormalize with two metrics $g^{\pm}_{ab}$.
- Bimetric repulsive gravity not possible [1].
- ⇒ Solution: *Multi*metric gravity.

Multimetric gravity

- $N \geq 3$ metric tensors g^I_{ab}.
- N standard model copies φ^I:

$$\Rightarrow S_M[g^I, \varphi^I] = \int d^4x \sqrt{g^I} \mathcal{L}_M[g^I, \varphi^I]\,.$$

- SM copies couple only to their metrics:

$$\Rightarrow \quad S = S_G[g^1, \ldots, g^N] + \sum_{I=1}^{N} S_M[g^I, \varphi^I]\,.$$

- ⇒ Field equations: $K^I_{ab} = 8\pi T^I_{ab}$.
- Example gravitational part of the action:

$$S_G = \frac{1}{16\pi} \int d^4x \prod_{I=1}^{N} (g^I)^{\frac{1}{2N}} \left[\sum_{I=1}^{N} c_1 R^I + \sum_{I,J=1}^{N} c_2 g^{I\,ij} R^J{}_{ij} + F^2(S) \right].$$

- Free constant parameters c_1 and c_2.
- $F^2(S)$ quadratic in $S^{IJ} = \Gamma^I - \Gamma^J$.

References

[1] M. Hohmann and M. N. R. Wohlfarth, Phys. Rev. D **80** (2009) 104011 [arXiv:0908.3384 [gr-qc]].

[2] M. Hohmann and M. N. R. Wohlfarth, Phys. Rev. D **81** (2010) 104006 [arXiv:1003.1379 [gr-qc]].

[3] M. Hohmann and M. N. R. Wohlfarth, Phys. Rev. D **82** (2010) 084028 [arXiv:1007.4945 [gr-qc]].

[4] M. Hohmann, Phys. Rev. D **85** (2012) 084024 [arXiv:1105.2555 [gr-qc]].

[•] M. Hohmann, arXiv:1309.7787 [gr-qc].

Summary

- Multimetric extension of PPN formalism.
- Application to concrete gravity theory.
- Experimental consistency confirmed.
- Consequence: accelerating expansion.
- Negative gravitational lensing.

Multimetric PPN formalism

- Assume perfect fluid matter.
- Introduce velocity orders $\mathcal{O}(n) \sim |\vec{v}|^n$:

$$g^I_{ab} = \eta_{ab} + h^{I(1)}_{ab} + h^{I(2)}_{ab} + h^{I(3)}_{ab} + h^{I(4)}_{ab} + \mathcal{O}(5)\,.$$

- Post-Newtonian metric ansatz:

$$h^{I(2)}_{00} = -\sum_{J=1}^{N} \alpha^{IJ} \triangle \chi^J\,,$$

$$h^{I(2)}_{\alpha\beta} = \sum_{J=1}^{N} \left(2\theta^{IJ} \chi^J{}_{,\alpha\beta} - (\gamma^{IJ} + \theta^{IJ}) \triangle \chi^J \delta_{\alpha\beta}\right),$$

$$h^{I(3)}_{0\alpha} = \sum_{J=1}^{N} \left(\sigma^{IJ}_{+} W^{J+}_{\alpha} + \sigma^{IJ}_{-} W^{J-}_{\alpha}\right),$$

$$h^{I(4)}_{00} = \sum_{J=1}^{N} \left(\phi^{IJ}_p \Phi^J_p + \phi^{IJ}_{\mathrm{II}} \Phi^J_{\mathrm{II}} + \sum_{A=1}^{2} \omega^{IJ}_A \Omega^J_A\right) + \sum_{J,K=1}^{N} \sum_{A=1}^{7} \psi^{IJK}_A \Psi^{JK}_A\,.$$

- Post-Newtonian potentials:

$$\chi^I, W^{I\pm}, \Phi^I_p, \Phi^I_{\mathrm{II}}, \Omega^I_1, \Omega^I_2, \Psi^{IJ}_1, \ldots, \Psi^{IJ}_7\,.$$

- Constant post-Newtonian parameters:

$$\alpha^{IJ}, \gamma^{IJ}, \theta^{IJ}, \sigma^{IJ}_{\pm}, \phi^{IJ}_p, \phi^{IJ}_{\mathrm{II}}, \omega^{IJ}_1, \omega^{IJ}_2, \psi^{IJK}_1, \ldots, \psi^{IJK}_7\,.$$

- Determined by field equations.
- Linked to observable effects:
  - Gravitational lensing.
  - Lorentz invariance.
  - Conservation laws.

Standard PPN formalism

- Identify visible matter with φ^1.
- ⇒ Matter distribution yields PPN potentials.
- Identify visible metric with g^1_{ab}.
- ⇒ g^1_{ab} observable in solar system tests.
- Consistency with experiments:

$$\alpha^{11} = 1\,, \quad \gamma^{11} = 1\,,$$
$$\sigma^{11}_{+} = -2\,, \quad \sigma^{11}_{-} = -\frac{3}{2}\,,$$
$$\phi^{11}_{\mathrm{II}} = 2\,, \quad \phi^{11}_p = 6\,,$$
$$\omega^{11}_1 = 4\,, \quad \omega^{11}_2 = 0\,,$$
$$\psi^{111}_1 = 0\,, \quad \psi^{111}_2 = 0\,,$$
$$\psi^{111}_3 = -2\,, \quad \psi^{111}_4 = 0\,,$$
$$\psi^{111}_5 = 0\,, \quad \psi^{111}_6 = -2\,,$$
$$\psi^{111}_7 = 0\,, \quad \theta^{11} = 0\,.$$

- Rescaling of Newtonian constant $\alpha^{11} = 1$.
- Gauge freedom $\theta^{11} = 0$ and $\psi^{111}_1 = \psi^{111}_5$.
- ⇒ 13 physical parameters can be measured.

Outlook

- Effects of new PPN potentials?
- How to measure new PPN parameters?
- Multimetric conservation laws?
- Tests using double pulsars?
- Tests using gravitational waves [4]?
- Cosmological tests / CMB?
- Simulations of structure formation?
- Search for dark galaxies?

Results

- Consistency conditions on c_1 and c_2:

$$c_1 + c_2 = \frac{1}{2-N}\,.$$

- Further conditions on $F^2(S)$.
- Multimetric PPN parameters:

$$\alpha^{IJ} = \gamma^{IJ} = 2\delta^{IJ} - 1\,, \quad \theta^{IJ} = 0\,,$$
$$\sigma^{IJ}_{+} = 2 - 4\delta^{IJ}\,, \quad \sigma^{IJ}_{-} = \frac{5}{2} - 4\delta^{IJ}\,,$$
$$\omega^{IJ}_1 = 10\delta^{IJ} - 6\,, \quad \omega^{IJ}_2 = 2 - 2\delta^{IJ}\,,$$
$$\phi^{IJ}_p = 16\delta^{IJ} - 10\,, \quad \phi^{IJ}_{\mathrm{II}} = 4\delta^{IJ} - 2$$

+ lengthy result for $\psi^{IJK}_1, \ldots, \psi^{IJK}_7$.

⇒ Multimetric gravity satisfies PPN bounds.

Cosmological consequences

- Cosmological symmetry:

$$g^I = -(n^I)^2(t) dt \otimes dt + (a^I)^2(t) \gamma_{\alpha\beta} dx^\alpha \otimes dx^\beta\,.$$

- Perfect fluid matter:

$$T^{Iab} = (\rho^I + p^I) u^{Ia} u^{Ib} + p^I g^{Iab}\,.$$

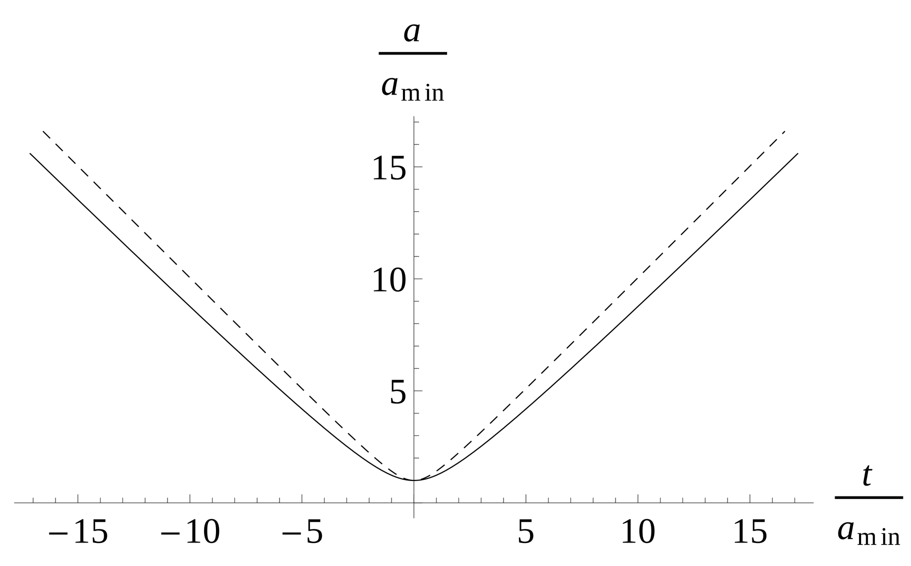
- Sector symmetry: drop all indices I.
- ⇒ Field equations simplify:

$$(c_1 + c_2)\left(R_{ab} - \frac{1}{2} R g_{ab}\right) = 8\pi T_{ab}\,.$$

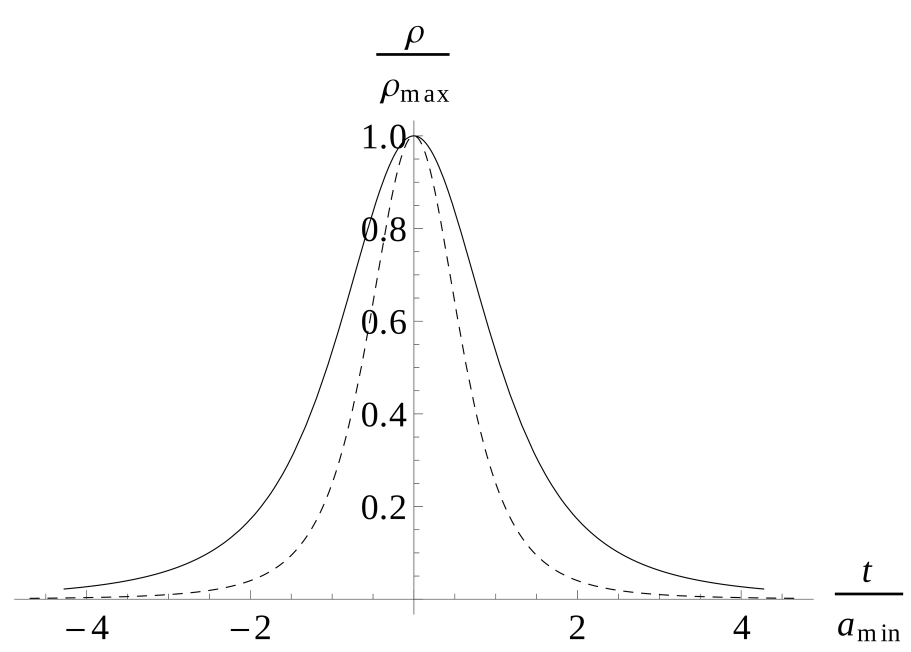
- ⇒ Accelerating expansion [2]:

$$\frac{\ddot{a}}{a} = \frac{4\pi}{3}(N-2)(\rho + 3p) \geq 0\,.$$

- Scale factor $a(t)$:



- Matter density $\rho(t)$:



- Hubble parameter $H = \dot{a}/a$: