Geometric constructions for multimetric repulsive gravity

Cosmology, structure formation and solar system physics

Manuel Hohmann

II. Institut für theoretische Physik

Universität Hamburg
DER FORSCHUNG | DER LEHRE | DER BILDUNG

Disputation
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Outline

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Einstein gravity

- Gravity is described by metric tensor g_{ab}.
- Einstein-Hilbert action:

$$S_G = \frac{1}{2} \int \omega R \, .$$

  - Volume form ω.
  - Scalar curvature R.
- Minimally coupled matter action:

$$S_M = \int \omega \mathcal{L}_M \, .$$

- Einstein equations:

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} \, .$$

Application to the universe

- 4.6% visible matter.

  [Komatsu *et al.* ’09]

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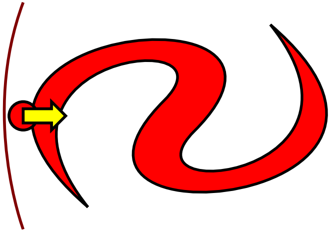

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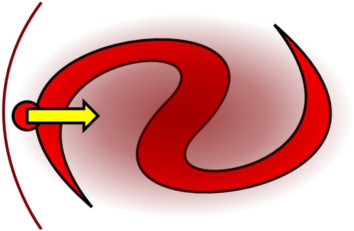

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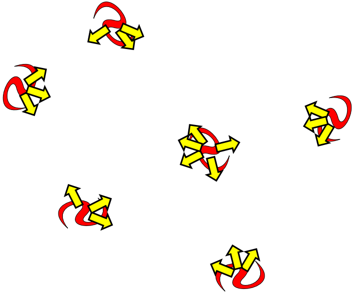

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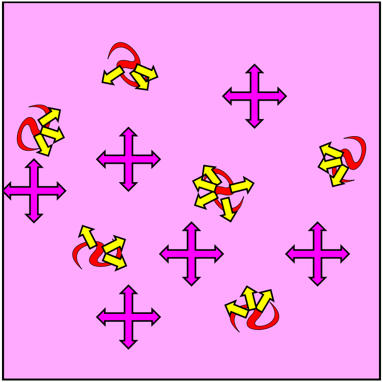

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$\Rightarrow$ Problem: What are dark matter and dark energy?

Explanations for the dark universe

- Particle physics:
  - Dark matter: [Bertone, Hooper, Silk '05]
    - Weakly interacting massive particles (WIMPs). [Ellis *et al.* '84]
    - Axions. [Preskill, Wise, Wilczek '83]
    - Massive compact halo objects (MACHOs). [Paczynski '86]
  - Dark energy: [Copeland, Sami, Tsujikawa '06]
    - Quintessence. [Peebles, Ratra '88]
    - K-essense. [Chiba, Okabe, Yamaguchi '00; Armendariz-Picon, Mukhanov, Steinhardt '01]
    - Chaplygin gas. [Kamenshchik, Moschella, Pasquier '01]
- Gravity:
  - Modified Newtonian dynamics (MOND). [Milgrom '83]
  - Tensor-vector-scalar theories. [Bekenstein '04]
  - Curvature corrections. [Schuller, Wohlfarth '05; Sotiriou, Faraoni '05]
  - Dvali-Gabadadze-Porrati (DGP) model. [Dvali, Gabadadze, Porrati '00, Lue '06]
  - Non-symmetric gravity. [Moffat '95]
  - Area metric gravity. [Punzi, Schuller, Wohlfarth '07]

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  - New idea: repulsive gravity $\Leftrightarrow$ negative mass!

Mass in Newtonian gravity

- Three types of mass! [Bondi '57]
  - Active gravitational mass m_a - source of gravity: $\phi = -G_N \frac{m_a}{r}$.
  - Passive gravitational mass m_p - reaction on gravity: $\vec{F} = -m_p \vec{\nabla}\phi$.
  - Inertial mass m_i - relates force to acceleration: $\vec{F} = m_i \vec{a}$.

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- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.

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- Gravity is always attractive.
- Convention: unit ratios and signs such that $m_a = m_p = m_i > 0$.
- Observations exist for visible mass only.

Dark universe from negative mass

- Idea for dark universe: standard model with $m_a = m_p = -m_i < 0$.
- Both copies couple only through gravity $\Rightarrow$ “dark”.
- Preserves momentum conservation.
- Breaks weak equivalence principle only for cross-interaction.

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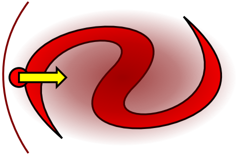

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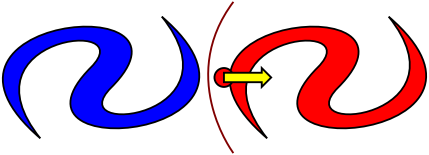

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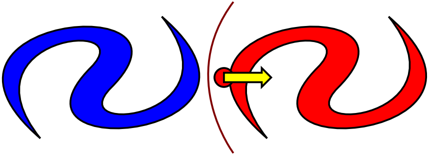
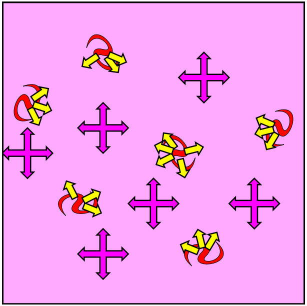


- Explanation of dark energy.



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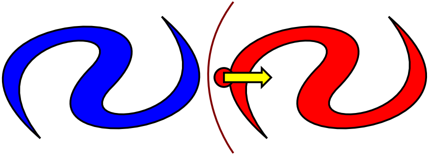

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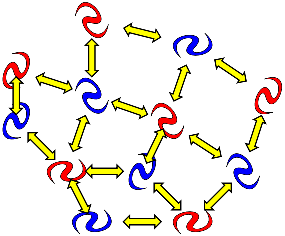

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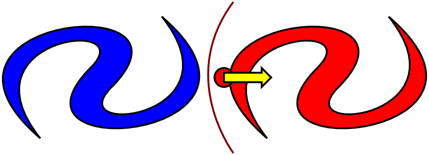

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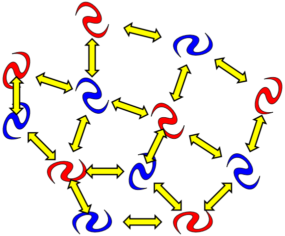

$\Rightarrow$ Advantage: Dark copy Ψ^- of well-known standard model Ψ^+:
  - No new parameters.
  - No unknown masses.
  - No unknown couplings.

Repulsive Einstein gravity

- Positive and negative test masses follow different trajectories.
- Two types of test mass trajectories $\Rightarrow$ two types of observers.
- Observer trajectories are autoparallels of two connections $\nabla^{\pm}$.
- Observers attach parallely transported frames to their curves.
- Frames are orthonormalized using two metric tensors $g^{\pm}_{ab}$.
- More general: N metrics g^{I}_{ab} and standard model copies Ψ^{I}.

Outline

Assumptions of the no-go theorem

1. Field content: standard model copies $\Psi^{\pm}$, metrics $g^{\pm}_{ab}$.
2. Gravitational field equations:

$$\underline{K}_{ab}[g^{+}, g^{-}] = \underline{M}_{ab}[g^{+}, g^{-}, \Psi^{+}, \Psi^{-}]\,.$$

3. Geometry: $\underline{K}_{ab}$ with at most second derivatives of $g^{\pm}$.
4. Matter source: $\underline{M}_{ab} = \underline{\underline{J}} \cdot \underline{T}_{ab}$ with $\underline{\underline{J}}$ invertible.
5. Vacuum solution: $g^{\pm}_{ab} = \lambda^{\pm}\eta_{ab}$ with $\lambda^{\pm} > 0$.
6. Post-Newtonian limit for non-moving dust:

$$g^{\pm} = \lambda^{\pm}\left[-(1 + 2I^{\pm}_{1})\,dt \otimes dt + (1 - 2I^{\pm}_{2})\delta_{\alpha\beta}dx^{\alpha} \otimes dx^{\beta}\right].$$

with gauge invariant post-Newtonian potentials $\underline{I}_2 = \underline{\underline{\gamma}} \cdot \underline{I}_1$.

No-go theorem

Theorem

We assume a bimetric theory with positive and negative mass sources and observers satisfying the assumptions detailed above. It is not possible to achieve a Newtonian limit with antisymmetric mass mixing in the Poisson equations for the vector $\underline{I}_1$ of gauge-invariant Newtonian potentials,

$$\triangle \underline{I}_1 = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \underline{\rho}\,.$$

[MH, M. Wohlfarth '09]

Proof: Field equations

- Linearization ansatz:

$$g^{\pm}_{ab} = \lambda^{\pm}(\eta_{ab} + h^{\pm}_{ab}) .$$

- Most general form of the linearized field equations:

$$\underline{K}_{ab} = \underline{\underline{P}} \cdot \partial^p \partial_{(a} \underline{h}_{b)p} + \underline{\underline{Q}} \cdot \Box \underline{h}_{ab} + \underline{\underline{R}} \cdot \partial_a \partial_b \underline{h} + \underline{\underline{M}} \cdot \partial_p \partial_q \underline{h}^{pq} \eta_{ab} + \underline{\underline{N}} \cdot \Box \underline{h} \eta_{ab} = \underline{\underline{J}} \cdot \underline{T}_{ab} .$$

- Parameter matrices $\underline{\underline{P}}, \underline{\underline{Q}}, \underline{\underline{R}}, \underline{\underline{M}}, \underline{\underline{N}}, \underline{\underline{J}}$ determined by full theory.
- Coordinate independent proof $\Leftrightarrow$ gauge-invariant formalism.

  [Bardeen ’80; Stewart ’90; Malik, Wands ’09]

- Gauge invariants = physical degrees of freedom, e.g., $I^{\pm}_1$.

Proof: Contradiction

- Equation for gauge-invariant Newtonian potential $\underline{I_1}$:

$$-2\underline{\underline{Q}} \cdot (\underline{\underline{1}} + \underline{\underline{\gamma}}) \cdot \triangle \underline{I_1} = \underline{\underline{J}} \cdot \underline{\underline{\lambda}} \cdot \underline{\rho}\,.$$

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- Two possible cases:
  1. $\underline{\underline{Q}} \cdot (\underline{\underline{1}} + \underline{\underline{\gamma}})$ is not invertible:
     LHS does not span $\mathbb{R}^2$, RHS does span $\mathbb{R}^2$!

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     LHS does not span $\mathbb{R}^2$, RHS does span $\mathbb{R}^2$!
  2. $\underline{\underline{Q}} \cdot (\underline{\underline{1}} + \underline{\underline{\gamma}})$ is invertible:
     Compare with desired Poisson equation:

$$\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \underline{\rho} = \triangle \underline{I_1} = -\frac{1}{2}(\underline{\underline{1}} + \underline{\underline{\gamma}})^{-1} \cdot \underline{\underline{Q}}^{-1} \cdot \underline{\underline{J}} \cdot \underline{\underline{\lambda}} \cdot \underline{\rho}\,.$$

LHS is not invertible, RHS is invertible! $\square$

Possible ways around the theorem

- More general source terms $\Leftrightarrow$ modified matter action.
  $\Rightarrow$ Possible problems with causality!
- More general Poisson equation:

$$\triangle \underline{I}_1 = \frac{1}{2}\begin{pmatrix} 1 & -\alpha \\ -\alpha & 1 \end{pmatrix} \cdot \underline{\rho} \,.$$

  $\Rightarrow$ Additional free parameter α!
- $N > 2$ standard model copies and metrics:

$$\triangle \underline{I}_1 = \frac{1}{2}\begin{pmatrix} 1 & -1 & \cdots & -1 \\ -1 & 1 & & -1 \\ \vdots & & \ddots & \\ -1 & -1 & & 1 \end{pmatrix} \cdot \underline{\rho} \,.$$

  $\Rightarrow$ Multimetric gravity!

Outline

Construction principles

1. Each standard model copy Ψ^I couples only to its metric g^I.

$$\Rightarrow \quad S_M[g^I, \Psi^I] = \int \omega^I \mathcal{L}_M[g^I, \Psi^I] \,.$$

2. Different sectors couple only gravitationally.

$$\Rightarrow \quad S = S_G[g^1, \ldots, g^N] + \sum_{I=1}^{N} S_M[g^I, \Psi^I] \,.$$

3. Field equations contain at most second derivatives of the metrics.
4. Symmetric with respect to permutations of the sectors (g^I, Ψ^I).

Construction of the theory

- Gravitational action:

$$S_G[g^1, \dots, g^N] = \frac{1}{2} \int d^4x \sqrt{g_0} \sum_{I,J=1}^{N} (x + y\delta^{IJ}) g^{Iij} R^J_{ij} \,.$$

- Variation of the action:

$$\delta S = -\frac{1}{2} \sum_{I=1}^{N} \int d^4x \sqrt{g_0} \tilde{K}^{I\,ab} \delta g^I_{ab} + \frac{1}{2} \sum_{I=1}^{N} \int d^4x \sqrt{g^I}\, T^{I\,ab} \delta g^I_{ab} \,.$$

- Equations of motion:

$$T^I_{ab} = \sqrt{g_0/g^I}\, \tilde{K}^I_{ab} = K^I_{ab} \,.$$

Geometry tensor

$$
\begin{aligned}
K^I_{ab} = \sqrt{g_0/g^I}\Big[&- \frac{1}{2N} g^I_{ab} \sum_{J,K=1}^{N} (x + y\delta^{JK}) g^{Jij} R^K{}_{ij} \\
&+ \sum_{J=1}^{N} (x + y\delta^{IJ}) R^J{}_{ab} - \Big(2\delta^d_{(a} g^I_{b)(i} \delta^c_{j)} - g^I_{ab} \delta^c_{(i} \delta^d_{j)} - g^{Icd} g^I_{i(a} g^I_{b)j} \Big) \times \\
&\times \sum_{J=1}^{N} (x + y\delta^{IJ}) \Big(2 g^{Jpi} S^{IJj}{}_{p(c} \tilde{S}^I{}_{d)} + \frac{1}{2} g^{Jij} \tilde{S}^I{}_c \tilde{S}^I{}_d + \frac{1}{2} g^{Jij} \nabla^I_c \tilde{S}^I{}_d \\
&+ \nabla^I_c S^{IJi}{}_{dp} g^{Jjp} + S^{IJp}{}_{cq} S^{IJi}{}_{dp} g^{Jjq} + S^{IJi}{}_{cq} S^{IJj}{}_{dp} g^{Jpq} \Big) \Big].
\end{aligned}
$$

Geometry tensor

$$K^I_{ab} = \sqrt{g_0/g^I}\Big[-\frac{1}{2N} g^I_{ab} \sum_{J,K=1}^{N} (x + y\delta^{JK}) g^{Jij} {R^K}_{ij}$$

$$+ \sum_{J=1}^{N} (x + y\delta^{IJ}) {R^J}_{ab} - \Big(2\delta^d_{(a} g^I_{b)(i}\delta^c_{j)} - g^I_{ab}\delta^c_{(i}\delta^d_{j)} - g^{Icd} g^I_{i(a} g^I_{b)j} \Big) \times$$

$$\times \sum_{J=1}^{N} (x + y\delta^{IJ}) \Big(2g^{Jpi} {S^{IJj}}_{p(c} {\tilde{S}^I}_{d)} + \frac{1}{2} g^{Jij} {\tilde{S}^I}_c {\tilde{S}^I}_d + \frac{1}{2} g^{Jij} \nabla^I_c {\tilde{S}^I}_d$$

$$+ \nabla^I_c {S^{IJi}}_{dp} g^{Jjp} + {S^{IJp}}_{cq} {S^{IJi}}_{dp} g^{Jjq} + {S^{IJi}}_{cq} {S^{IJj}}_{dp} g^{Jpq} \Big) \Big].$$

Ricci tensors.

$$K^I_{ab} = \sqrt{g_0/g^I}\Bigg[-\frac{1}{2N} g^I_{ab} \sum_{J,K=1}^{N} (x + y\delta^{JK}) g^{Jij} R^K{}_{ij}$$

$$+ \sum_{J=1}^{N} (x + y\delta^{IJ}) R^J{}_{ab} - \left(2\delta^d_{(a} g^I_{b)(i} \delta^c_{j)} - g^I_{ab}\delta^c_{(i}\delta^d_{j)} - g^{Icd} g^I_{i(a} g^I_{b)j} \right) \times$$

$$\times \sum_{J=1}^{N} (x + y\delta^{IJ}) \Big(2g^{Jpi} S^{IJj}{}_{p(c} \tilde{S}^I{}_{d)} + \frac{1}{2} g^{Jij} \tilde{S}^I{}_c \tilde{S}^I{}_d + \frac{1}{2} g^{Jij} \nabla^I_c \tilde{S}^I{}_d$$

$$+ \nabla^I_c S^{IJi}{}_{dp} g^{Jjp} + S^{IJp}{}_{cq} S^{IJi}{}_{dp} g^{Jjq} + S^{IJi}{}_{cq} S^{IJj}{}_{dp} g^{Jpq} \Big) \Bigg] .$$

Connection difference tensors (first derivative order):

$$S^{IJi}{}_{jk} = \Gamma^{Ii}{}_{jk} - \Gamma^{Ji}{}_{jk} , \qquad S^{IJ}{}_j = S^{IJk}{}_{jk} ,$$

$$\tilde{S}^{Ji}{}_{jk} = \frac{1}{N} \sum_{I=1}^{N} S^{IJi}{}_{jk} , \qquad \tilde{S}^J{}_j = \tilde{S}^{Jk}{}_{jk} .$$

Newtonian limit

- Calculate Poisson equation:

$$\triangle I_1^I = \frac{2}{3}\left(Nx - y\right)^{-1} \sum_{J=1}^{N} \left(\frac{7Nx + y}{4N(Nx + y)} - \delta^{IJ}\right)\rho^J \,.$$

- Antisymmetric gravitational forces for parameter values

$$x = \frac{2N - 1}{6N(2 - N)}\,, \quad y = \frac{-2N + 7}{6(2 - N)}\,.$$

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- Antisymmetric gravitational forces for parameter values

$$x = \frac{2N - 1}{6N(2 - N)}, \quad y = \frac{-2N + 7}{6(2 - N)} .$$

- Three different cases:
  - $N = 1$ reduces to Einstein gravity.
  - $N = 2$ is excluded.
  - $N \geq 3$ is the desired repulsive gravity theory. [MH, M. Wohlfarth '10]

Outline

Cosmological symmetry

- Standard cosmology: Robertson–Walker metrics

$$g^I = -n_I^2(t)dt \otimes dt + a_I^2(t)\gamma_{\alpha\beta}dx^\alpha \otimes dx^\beta \,.$$

  - Lapse functions n_I.
  - Scale factors a_I.
  - Spatial metric $\gamma_{\alpha\beta}$ of constant curvature $k \in \{-1, 0, 1\}$ and Riemann tensor $R(\gamma)_{\alpha\beta\gamma\delta} = 2k\gamma_{\alpha[\gamma}\gamma_{\delta]\beta}$.
- Perfect fluid matter:

$$T^{I\,ab} = (\rho_I + p_I)u^{Ia}u^{Ib} + p_I g^{Iab} \,.$$

- Normalization: $g^I_{ab}u^{Ia}u^{Ib} = -1$.

Simple cosmological model

- Early universe: radiation; late universe: dust.
- Copernican principle: common evolution for all matter sectors.

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- Early universe: radiation; late universe: dust.
- Copernican principle: common evolution for all matter sectors.

$\Rightarrow$ Single effective energy-momentum tensor $T^I_{ab} = T_{ab}$.

$\Rightarrow$ Single effective metric $g^I_{ab} = g_{ab}$.

$\Rightarrow$ Common scale factors $a^I = a$ and lapse functions $n^I = n$.

$\Rightarrow$ Rescale cosmological time to set $n \equiv 1$.

$\Rightarrow$ Ricci tensors $R^I_{ab} = R_{ab}$ become equal.

$\Rightarrow$ Connection differences $S^{IJ\,i}{}_{jk} = 0$ vanish.

Simple cosmological model

- Early universe: radiation; late universe: dust.
- Copernican principle: common evolution for all matter sectors.

⇒ Single effective energy-momentum tensor $T^I_{ab} = T_{ab}$.

⇒ Single effective metric $g^I_{ab} = g_{ab}$.

⇒ Common scale factors $a^I = a$ and lapse functions $n^I = n$.

⇒ Rescale cosmological time to set $n \equiv 1$.

⇒ Ricci tensors $R^I_{ab} = R_{ab}$ become equal.

⇒ Connection differences $S^{IJ\,i}{}_{jk} = 0$ vanish.

⇒ Equations of motion simplify:

$$(2 - N)T_{ab} = R_{ab} - \frac{1}{2}Rg_{ab}\,.$$

⇒ Negative effective gravitational constant for early / late universe.

Cosmological equations of motion

- Insert Robertson–Walker metric into equations of motion:

$$\rho = \frac{3}{2-N}\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right),$$
$$p = -\frac{1}{2-N}\left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right).$$

$\Rightarrow$ Positive matter density $\rho > 0$ requires $k = -1$ and $\dot{a}^2 < 1$.

$\Rightarrow$ No solutions for $k = 0$ or $k = 1$.

Accelerating expansion

- Acceleration equation:

$$\frac{\ddot{a}}{a} = \frac{N-2}{6}(\rho + 3p)\,.$$

- Factor $N - 2 > 0$ for multimetric gravity.
- Strong energy condition

$$\left(T_{ab} - \frac{1}{2}Tg_{ab}\right)t^a t^b \geq 0$$

for all timelike vector fields t^a implies $\rho + 3p \geq 0$.

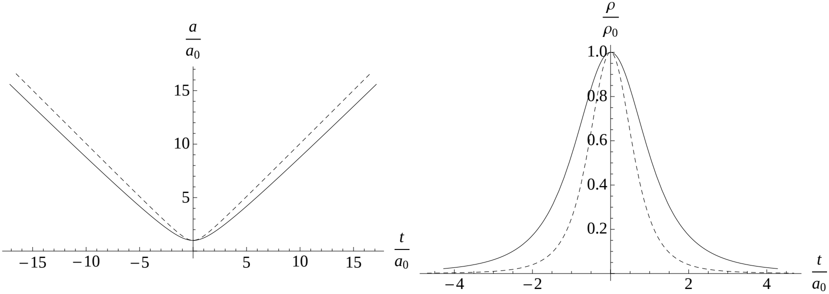
$\Rightarrow$ Acceleration must be positive.

Explicit solution

- Equation of state: $p = \omega\rho$; dust: $\omega = 0$, radiation: $\omega = 1/3$.
- General solution using conformal time η defined by $dt = a\,d\eta$:

$$a = a_0 \left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{\frac{2}{3\omega+1}} ,$$

$$\rho = \frac{3}{(N-2)a_0^2}\left(\cosh\left(\frac{3\omega+1}{2}(\eta-\eta_0)\right)\right)^{-\frac{6\omega+6}{3\omega+1}} .$$



$\Rightarrow$ Big bounce at $\eta = \eta_0$. [MH, M. Wohlfarth '10]

Outline

Ingredients

1. Metrics $g^I_{ab} = g^0_{ab} + h^I_{ab}$ with
$$g^0 = -dt \otimes dt + a^2(t)\gamma_{\alpha\beta} dx^\alpha \otimes dx^\beta$$
and $a(t)$ determined by cosmology.
2. Scale for structure formation $\ll$ curvature radius of the universe:
   - Cubic volume $0 \leq x^\alpha \leq \ell$.
   - Approximate $\gamma_{\alpha\beta}$ by $\delta_{\alpha\beta}$.
   - Periodic boundary conditions.
3. Matter content: n point masses M for each sector.
   - Model for dust matter: $p = 0$.
   - Matter density:
$$\rho = \frac{Mn}{(a\ell)^3} \, .$$
4. Large mean distance $a\ell/\sqrt[3]{Nn} \gg 2GM$.
5. Small velocities $|v^\alpha_{Ii}| = |a\dot{x}^\alpha_{Ii}| \ll 1$.

Local dynamics

- Masses of type I follow geodesics of their metric g^I_{ab}:

$$\ddot{x}^\alpha_{Ii} = \frac{\partial_\alpha h^I_{00}}{2a^2} - 2\frac{\dot{a}}{a}\dot{x}^\alpha_{Ii} \,.$$

- Antisymmetric Poisson equation:

$$h^I_{00} = -2\sum_{J=1}^{N}(2\delta^{IJ} - 1)\Phi^J \,.$$

- Individual Newtonian potentials $\Phi^I(t,\vec{x})$:

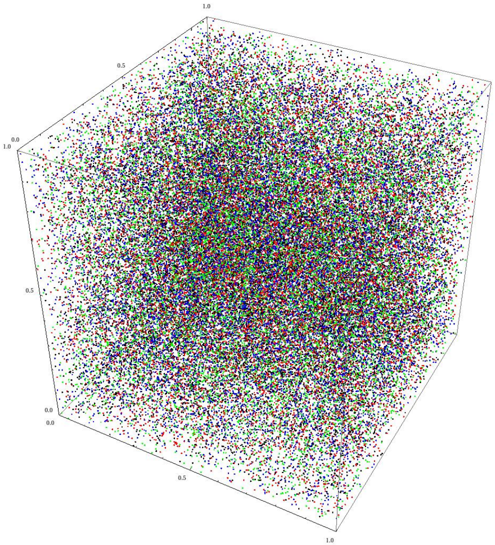
$$\Phi^I(t,\vec{x}) = -\frac{M}{a(t)}\sum_{i=1}^{n}\frac{1}{d(\vec{x},\vec{x}_{Ii}(t))} \,.$$

- Periodic distance function $d(\vec{x},\vec{x'})$:

$$d(\vec{x},\vec{x'}) = \min_{\vec{k}\in\mathbb{Z}^3}\left|\vec{x} - \vec{x}' + \ell\vec{k}\right| \,.$$

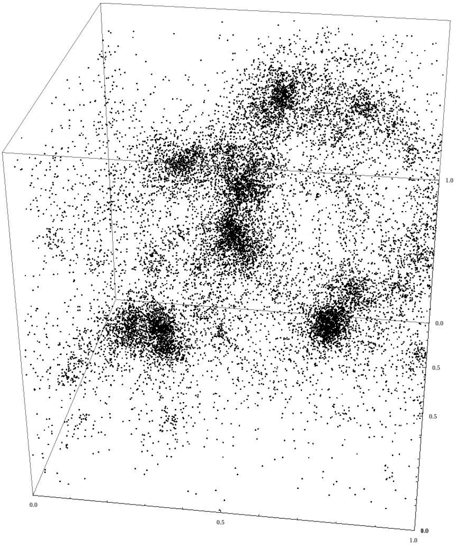
Evolution of structures for all matter types

- $N = 4$.
- $n = 16384$.
- 7.5 days CPU time.



Final state for one matter type

- Galactic clusters.
- Empty voids.



Outline

Physical situation in the solar system

1. Consider only repulsive gravity between different sectors.
   - $\Rightarrow$ Different matter types should separate.
   - $\Rightarrow$ Energy-momentum tensor contains only visible matter:
     - $\Rightarrow$ $T^{+}_{ab} = T^{1}_{ab} \neq 0$.
     - $\Rightarrow$ $T^{-}_{ab} = T^{2}_{ab} = \ldots = T^{N}_{ab} = 0$.

Physical situation in the solar system

1. Consider only repulsive gravity between different sectors.
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     - $\Rightarrow$ $T^-_{ab} = T^2_{ab} = \ldots = T^N_{ab} = 0$.
2. Permutation symmetry between sectors.
   - $\Rightarrow$ Visible matter has equal effects on all dark sectors.
   - $\Rightarrow$ Metric:
     - $\Rightarrow$ $g^+_{ab} = g^1_{ab}$.
     - $\Rightarrow$ $g^-_{ab} = g^2_{ab} = \ldots = g^N_{ab}$.

Parametrized post-Newtonian formalism

- Obtain "fingerprint" of single-metric gravity theories. [Thorne, Will '71; Will '93]
- Expansion of the metric in velocity orders:

$$g_{ab} = g^0_{ab} + h_{ab} = g^0_{ab} + h^{(1)}_{ab} + h^{(2)}_{ab} + h^{(3)}_{ab} + h^{(4)}_{ab}\,.$$

- Decomposition of h_{ab}:
  - PPN potentials: $U, V_\alpha, W_\alpha, \Phi_W, \Phi_1 \ldots \Phi_4, \mathcal{A}, \not{\mathcal{B}}, \not{U}_{\alpha\beta}$.
  - PPN parameters: $\not{\alpha}, \gamma, \not{\theta}, \sigma_\pm, \beta, \xi, \phi_1 \ldots \phi_4, \mu, \not{\psi}$.
- Expand equations of motion:
  - up to quadratic order in h_{ab},
  - up to fourth velocity order.
- Solve equations of motion order by order $\Rightarrow$ PPN parameters.
- Linearized field equations already determine γ, σ_+.
- Values constrained by experiment, e.g., $\gamma = 1 \pm 2.3 \cdot 10^{-5}$.

Parametrized post-Newtonian formalism

- Obtain "fingerprint" of multimetric gravity theories. [MH, M. Wohlfarth '10]
- Expansion of the metric in velocity orders:

$$g^{\pm}_{ab} = g^{0}_{ab} + h^{\pm}_{ab} = g^{0}_{ab} + h^{\pm(1)}_{ab} + h^{\pm(2)}_{ab} + h^{\pm(3)}_{ab} + h^{\pm(4)}_{ab} \,.$$

- Decomposition of $h^{\pm}_{ab}$:
  - PPN potentials: $U, V_\alpha, W_\alpha, \Phi_W, \Phi_1 \ldots \Phi_4, \mathcal{A}, \mathcal{B}, U_{\alpha\beta}$.
  - PPN parameters: $\alpha^{\pm}, \gamma^{\pm}, \theta^{\pm}, \sigma^{\pm}_{\pm}, \beta^{\pm}, \xi^{\pm}, \phi^{\pm}_1 \ldots \phi^{\pm}_4, \mu^{\pm}, \nu^{\pm}$.
- Expand equations of motion:
  - up to quadratic order in $h^{\pm}_{ab}$,
  - up to fourth velocity order.
- Solve equations of motion order by order $\Rightarrow$ PPN parameters.
- Linearized field equations already determine $\alpha^{\pm}, \gamma^{\pm}, \theta^{\pm}, \sigma^{\pm}_{+}$.
- Values constrained by experiment, e.g., $\gamma^{+} = 1 \pm 2.3 \cdot 10^{-5}$.

PPN parameters

- Apply linearized PPN formalism to presented theory:

$$
\begin{aligned}
\alpha^{+} &= 1\,, & \alpha^{-} &= -1\,,\\
\gamma^{+} &= \frac{1}{N}\,, & \gamma^{-} &= \frac{3}{2N-7} + \frac{1}{N} + \frac{1}{2}\,,\\
\theta^{+} &= 0\,, & \theta^{-} &= \frac{1}{7-2N} - \frac{3}{2}\,,\\
\sigma^{+}_{+} &= -1 - \frac{1}{N}\,, & \sigma^{-}_{+} &= 2 - \frac{1}{N}\,.
\end{aligned}
$$

PPN parameters

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$$\alpha^+ = 1\,, \qquad \alpha^- = -1\,,$$
$$\gamma^+ = \frac{1}{N}\,, \qquad \gamma^- = \frac{3}{2N-7} + \frac{1}{N} + \frac{1}{2}\,,$$
$$\theta^+ = 0\,, \qquad \theta^- = \frac{1}{7-2N} - \frac{3}{2}\,,$$
$$\sigma^+_+ = -1 - \frac{1}{N}\,, \qquad \sigma^-_+ = 2 - \frac{1}{N}\,.$$

- $\alpha^+ = -\alpha^- = 1$ corresponds to repulsive Newtonian limit.

PPN parameters

- Apply linearized PPN formalism to presented theory:

$$
\begin{aligned}
1 &= \alpha^+ = 1\,, & \alpha^- &= -1\,,\\
1 &= \gamma^+ = \frac{1}{N}\,, & \gamma^- &= \frac{3}{2N-7} + \frac{1}{N} + \frac{1}{2}\,,\\
0 &= \theta^+ = 0\,, & \theta^- &= \frac{1}{7-2N} - \frac{3}{2}\,,\\
-2 &= \sigma^+_+ = -1 - \frac{1}{N}\,, & \sigma^-_+ &= 2 - \frac{1}{N}\,.
\end{aligned}
$$

- $\alpha^+ = -\alpha^- = 1$ corresponds to repulsive Newtonian limit.
- Experimentally measured values are obtained only for $N = 1$.

Improved PPN consistent theory

- Add correction term to the gravitational action:

$$\bar{S}_G = \frac{1}{2}\sum_{I=1}^{N}\int d^4x\sqrt{g_0}\,g^{I\,ij}\left(z\tilde{S}^I{}_k\tilde{S}^{I\,k}{}_{ij} + u\tilde{S}^I{}_i\tilde{S}^I{}_j\right).$$

- PPN consistency and repulsive Newtonian limit are achieved for:

$$x = \frac{1}{8-4N}\,, \quad y = \frac{4-N}{8-4N}\,, \quad z = -\frac{4-N}{8-4N}\,, \quad u = -\frac{12-3N}{8-4N}\,.$$

- PPN parameters:

$$\begin{array}{llll} \alpha^+ = 1\,, & \gamma^+ = 1\,, & \theta^+ = 0\,, & \sigma^+_+ = -2\,, \\ \alpha^- = -1\,, & \gamma^- = -1\,, & \theta^- = 0\,, & \sigma^-_+ = 2\,. \end{array}$$

- Correction terms do not affect cosmology or structure formation.

Outline

Summary

- General relativity describes visible universe.
- Fit to observations requires dark matter & dark energy.
- Idea here: Dark universe might be explained by repulsive gravity.

$\Rightarrow$ Repulsive gravity requires an extension of general relativity.

$\Rightarrow$ No-go theorem: bimetric repulsive gravity is not possible.

$\Rightarrow$ Multimetric repulsive gravity with $N \geq 3$ by explicit construction.

$\Rightarrow$ Cosmology features late-time acceleration and big bounce.

$\Rightarrow$ Structure formation features clusters and voids.

$\Rightarrow$ Repulsive gravity is consistent with PPN bounds.

Outlook

- Remaining PPN parameters should be determined from full multimetric PPN formalism.
- Restrict multimetric gravity theories by additional PPN bounds.
- Establish further construction principles, e.g., continuous symmetry between sectors.
- Examine initial-value problem.
- Determine further exact solutions (single point mass. . .).
- Advanced simulation of structure formation including thermodynamics using GADGET-2 (Millenium Simulation).
- Search for repulsive gravity sources in the galactic voids through gravitational lensing.
- Application to binaries: gravitational radiation should be emitted in all sectors, but only one type is visible.

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- Application to binaries: gravitational radiation should be emitted in all sectors, but only one type is visible. Prediction!

Cauchy problem for multimetric gravity

- Linearized vacuum field equations:

$$C^{pqrs}{}_{ab}{}^{I}{}_{J}\partial_p\partial_q h^J_{rs} = 0\,.$$

- Rewrite as first order equations:

$$\partial_q h^I_{rs} - k^I_{qrs} = 0\,,$$
$$C^{pqrs}{}_{ab}{}^{I}{}_{J}\partial_p k^J_{qrs} = 0\,.$$

- Equations are of the form:

$$A^{p\,\psi}{}_{\varphi}\partial_p f^{\varphi} + B^{\psi}{}_{\varphi}f^{\varphi} = 0\,.$$

- Cauchy problem is well-posed if

$$P(q) = \det_{\psi\varphi}(A^{p\,\psi}{}_{\varphi}q_p)$$

is hyperbolic for all timelike covectors q.

Quantum manifolds: Concept

$$
\begin{tikzcd}
 & M_Q \arrow[dl, "\text{position expectation}"'] \arrow[dr, leftrightsquigarrow, "\text{local homeomorphism}"] & \\
M \arrow[dr, leftrightsquigarrow, "\text{local homeomorphism}"'] & & \mathcal{S} \arrow[dl, "\text{position expectation}"] \\
 & \mathbb{R}^n &
\end{tikzcd}
$$

- Classical mechanics: Euclidian space $\mathbb{R}^n$.
- Quantum mechanics: Schwartz space $\mathcal{S}$.
- General relativity: Differentiable manifold M.
- Quantum gravity: Quantum manifold M_Q? [MH, M. Wohlfarth ’08]

Quantum manifolds: Construction

$$
\begin{tikzcd}
M_Q \supset & U \arrow[r, "\phi"] \arrow[d, "\mathcal{Q}"'] & V \arrow[d, "\bar{\boldsymbol{Q}}"] & \subset \mathcal{S}^{\neq 0}(\mathbb{R}^n) \\
M \supset & X \arrow[r, "\chi"'] & W & \subset \mathbb{R}^n
\end{tikzcd}
$$

- Chart (U, ϕ) of M_Q.
- Position expectation value $\bar{\boldsymbol{Q}}$.
- Open set $V = \bar{\boldsymbol{Q}}^{-1}(W)$ for some open set W.
- Lift topology of $\mathbb{R}^n$ to M_Q via $\bar{\boldsymbol{Q}} \circ \phi$.

$\Rightarrow$ Non-Hausdorff topology on M_Q.

- Take Kolmogorov quotient $M = \mathcal{Q}(M_Q)$.

$\Rightarrow$ Hausdorff topology on M.

$\Rightarrow$ Unique homeomorphism χ such that $\chi \circ \mathcal{Q} = \bar{\boldsymbol{Q}} \circ \phi$.

$\Rightarrow$ (X, χ) is a chart of M.

Quantum manifolds: Differential structure

$$
\begin{tikzcd}
\phi_i(U_i \cap U_j) \arrow[rr, "\phi_{ji}"] \arrow[ddd, "\bar{\boldsymbol{Q}}"] & & \phi_j(U_i \cap U_j) \arrow[ddd, "\bar{\boldsymbol{Q}}"] \\
& U_i \cap U_j \arrow[ul, "\phi_i"'] \arrow[ur, "\phi_j"] \arrow[d, "\mathcal{Q}"] & \\
& X_i \cap X_j \arrow[dl, "\chi_i"] \arrow[dr, "\chi_j"] & \\
\chi_i(X_i \cap X_j) \arrow[rr, "\chi_{ji}"] \arrow[uuu, bend left, "\Psi"] & & \chi_j(X_i \cap X_j)
\end{tikzcd}
$$

- Two charts (U_i, ϕ_i) and (U_j, ϕ_j).
- Choose non-unique inverse

$$\Psi : \mathbb{R}^n \to \mathcal{S}^{\neq 0}(\mathbb{R}^n)\,,\ \boldsymbol{x} \mapsto \left(\boldsymbol{y} \mapsto e^{-(\boldsymbol{y}-\boldsymbol{x})^2}\right).$$

$\Rightarrow$ $\bar{\boldsymbol{Q}} \circ \phi_{ji} \circ \Psi = \chi_{ji} \circ \bar{\boldsymbol{Q}} \circ \Psi = \chi_{ji}$ is differentiable!

Quantum manifolds: Fiber bundle

$$
\begin{tikzcd}
U \arrow[rrr, "\phi"] \arrow[ddd, "\mathcal{Q}"] \arrow[dr, "\omega"] & & & V \arrow[ddd, "\bar{Q}"] \arrow[dl, "\tau"'] \\
& X \times \mathcal{S}_0 \arrow[r, "\chi\times\mathrm{id}_{\mathcal{S}_0}"] \arrow[ddl, "p_1"] & W \times \mathcal{S}_0 \arrow[ddr, "p_1"] & \\
& & & \\
X \arrow[rrr, "\chi"] & & & W
\end{tikzcd}
$$

- Chart (U, ϕ) of M_Q.
- V is trivial $\mathcal{S}_0$-bundle over W.
- Use χ to construct unique homeomorphism ω.
- U is trivial $\mathcal{S}_0$-bundle over X.
- M_Q is $\mathcal{S}_0$-bundle over M.

Quantum manifolds: Interpretation

- Quantization of fields:
  - Classical fields living on the quantum manifold M_Q.
  - Measurement at $\xi \in M$ randomly picks value at $\mathcal{Q}^{-1}(\xi) \in M_Q$.
  - Probabilities are defined by measure on M_Q.

  $\Rightarrow$ Quantum measurement.

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  - Probabilities are defined by measure on M_Q.

  $\Rightarrow$ Quantum measurement.
- Quantization of a point particle:
  - Point particle trajectory in M_Q.
  - Position measurement yields only projected position $\xi \in M$.
  - Full dynamics in M_Q is hidden to classical observer.

  $\Rightarrow$ Quantum dynamics.