

Parameterized post-Newtonian formalism for scalar-tensor gravity with a potential

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Scalar tensor gravity

- Fields: metric $g_{\mu\nu}$, scalar field Ψ , matter.
- Action:

$$S = S_m[g_{\mu\nu}, \chi_m] + \frac{1}{2\kappa^2} \int_{V_4} d^4x \sqrt{-g} \left(\Psi R - \frac{\omega(\Psi)}{\Psi} \partial_\rho \Psi \partial^\rho \Psi - 2\kappa^2 V(\Psi) \right).$$

- Metric field equation:

$$R_{\mu\nu} = \frac{1}{\Psi} \left[\kappa^2 (T_{\mu\nu} - \frac{\omega + 1}{2\omega + 3} g_{\mu\nu} T) + \nabla_\mu \partial_\nu \Psi + \frac{\omega}{\Psi} \partial_\mu \Psi \partial_\nu \Psi - \frac{g_{\mu\nu}}{4\omega + 6} \frac{d\omega}{d\Psi} \partial_\rho \Psi \partial^\rho \Psi \right] + \frac{\kappa^2}{\Psi} g_{\mu\nu} \frac{2\omega + 1}{2\omega + 3} V + g_{\mu\nu} \frac{\kappa^2}{2\omega + 3} \frac{dV}{d\Psi}$$

- Scalar field equation:

$$\square \Psi = \frac{1}{2\omega + 3} \left[\kappa^2 T - \frac{d\omega}{d\Psi} \partial_\rho \Psi \partial^\rho \Psi + 2\kappa^2 (\Psi \frac{dV}{d\Psi} - 2V) \right]$$

Post-Newtonian approximation

- Expansion around background solution:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \Psi = \Psi_0 + \psi.$$

- Taylor expansion:

$$\omega = \omega_0 + \omega_1 \psi + \mathcal{O}(\psi^2), \\ V = V_2 \psi^2 + V_3 \psi^3 + \mathcal{O}(\psi^4).$$

- Mass of the scalar field:

$$m_\psi = 2\kappa \sqrt{\frac{\Psi_0 V_2}{2\omega_0 + 3}}.$$

- Perfect fluid matter with density ρ , internal energy $\rho\Pi$, pressure p , velocity $v^i = \frac{u^i}{u^0}$:

$$T^{\mu\nu} = (\rho + \rho\Pi + p)u^\mu u^\nu + pg^{\mu\nu}.$$

- Velocity orders $\mathcal{O}(n) \sim |\vec{v}|^n$:

$$h_{00} = h_{00}^{(2)} + h_{00}^{(4)} + \mathcal{O}(6), \\ h_{0i} = h_{0i}^{(3)} + \mathcal{O}(5), h_{ij} = h_{ij}^{(2)} + \mathcal{O}(4), \\ \psi = \psi^{(2)} + \psi^{(4)} + \mathcal{O}(6).$$

- Gauge conditions:

$$h_{i,j}^j - \frac{1}{2} h_{\mu}^{\mu}{}_{,i} = \frac{1}{\Psi_0} \psi_{,a}, \\ h_{0,j}^j - \frac{1}{2} h_{j,j}^j{}_{,0} = \frac{1}{\Psi_0} \psi_{,0}.$$

Point mass approximation

- Matter source:

$$\rho = M\delta(\vec{x}), \Pi = 0, p = 0, v_i = 0.$$

- Newtonian potential:

$$U(r) = \frac{\kappa^2}{8\pi} \frac{M}{r}.$$

- Gravitational self-energy:

$$\Phi^{(4)} = 0.$$

Newtonian limit

- Scalar field:

$$\psi^{(2)}(r) = \frac{2}{2\omega_0 + 3} U(r) e^{-m_\psi r}.$$

- Metric perturbation:

$$h_{00}^{(2)}(r) = 2G_{\text{eff}}(r) U(r).$$

- Effective Newtonian constant:

$$G_{\text{eff}}(r) = \frac{1}{\Psi_0} \left(1 + \frac{e^{-m_\psi r}}{2\omega_0 + 3} \right).$$

PPN parameter $\gamma(r)$

- Metric perturbation:

$$h_{ij}^{(2)}(r) = 2G_{\text{eff}}(r) \gamma(r) U(r) \delta_{ij}.$$

- Solution for $\gamma(r)$:

$$\gamma(r) = \frac{2\omega_0 + 3 - e^{-m_\psi r}}{2\omega_0 + 3 + e^{-m_\psi r}}.$$

- L. Perivolaropoulos 2009.
- G. J. Olmo 2005.
- Y. Xie *et al.* 2007.

PPN parameter $\beta(r)$

- Scalar field:

$$\psi^{(4)}(r) = \frac{2}{(2\omega_0 + 3)^2} \left(\frac{1}{\Psi_0} - \frac{\omega_1}{2\omega_0 + 3} \right) U^2(r) e^{-2m_\psi r} \\ + \frac{2m_\psi}{\Psi_0(2\omega_0 + 3)} U^2(r) r (e^{m_\psi r} \text{Ei}(-2m_\psi r) - e^{-m_\psi r} \ln(m_\psi r)) \\ + \frac{3m_\psi}{(2\omega_0 + 3)^2} \left(\frac{V_3}{V_2} - \frac{1}{\Psi_0} - \frac{\omega_1}{2\omega_0 + 3} \right) U^2(r) r (e^{m_\psi r} \text{Ei}(-3m_\psi r) - e^{-m_\psi r} \text{Ei}(-m_\psi r)).$$

- Metric perturbation:

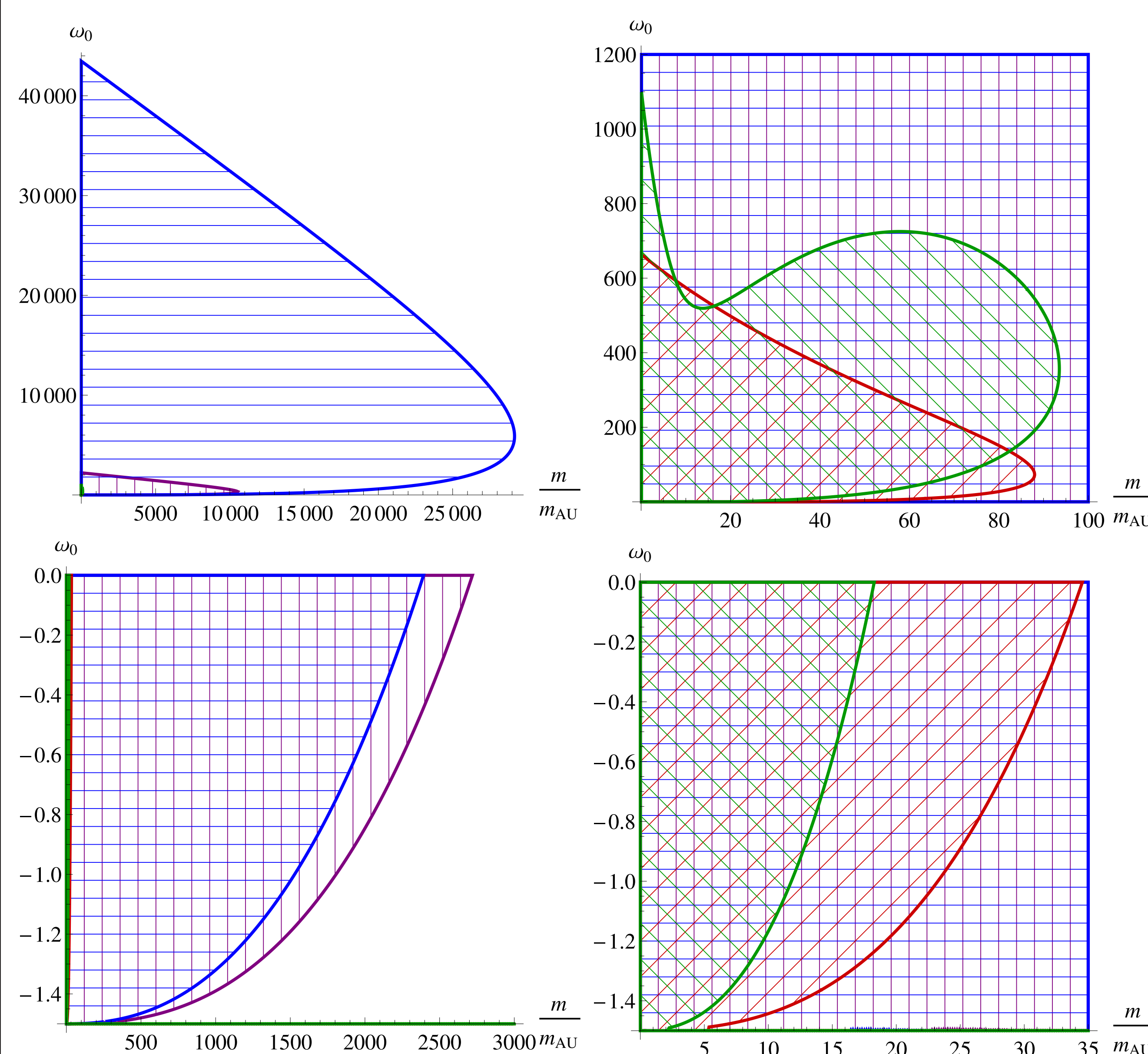
$$h_{00}^{(4)}(r) = -2G_{\text{eff}}^2(r) \beta(r) U^2(r) + \Phi^{(4)}(r).$$

- Potential $\Phi^{(4)}(r)$ absorbs terms $\sim \rho U$ coming from gravitational self-energy.
- Solution for $\beta(r)$:

$$\beta(r) = 1 + \frac{\omega_1 e^{-2m_\psi r}}{G_{\text{eff}}^2(r) \Psi_0 (2\omega_0 + 3)^3} - \frac{m_\psi r}{G_{\text{eff}}^2(r) (2\omega_0 + 3)} \left[\frac{2}{\Psi_0^2} [e^{-m_\psi r} + \text{Ei}(-m_\psi r) m_\psi r] \right. \\ + \frac{1}{2\Psi_0(2\omega_0 + 3)} \left(\frac{2\omega_0 - 2}{\Psi_0} + \frac{3V_3}{V_2} - \frac{\omega_1}{2\omega_0 + 3} \right) [e^{-2m_\psi r} + 2\text{Ei}(-2m_\psi r) m_\psi r] \\ + \frac{1}{\Psi_0^2} (2e^{-m_\psi r} - e^{m_\psi r} \text{Ei}(-2m_\psi r) + 2\text{Ei}(-m_\psi r) m_\psi r + e^{-m_\psi r} \ln(m_\psi r)) \\ + \frac{3}{2\Psi_0(2\omega_0 + 3)} \left(\frac{V_3}{V_2} - \frac{1}{\Psi_0} - \frac{\omega_1}{2\omega_0 + 3} \right) \\ \left. \times (-e^{-2m_\psi r} + e^{m_\psi r} \text{Ei}(-3m_\psi r) - 2\text{Ei}(-2m_\psi r) m_\psi r - e^{-m_\psi r} \text{Ei}(-m_\psi r)) \right].$$

- MH, LJ, PK, ER - to appear soon.

Comparison with experiments: lined regions ruled out



- Effect of ω_1, V_3 negligible.
- x -axis: mass parameter

$$m = 2\kappa \sqrt{\Psi_0 V_2}.$$

- y -axis: ω_0 .

- Very long baseline interferometry
 $r_0 \approx 4.65 \cdot 10^{-3}$ AU
 $\gamma - 1 = (-1.7 \pm 4.5) \cdot 10^{-4}$
- Cassini tracking
 $r_0 \approx 7.44 \cdot 10^{-3}$ AU
 $\gamma - 1 = (2.1 \pm 2.3) \cdot 10^{-5}$
- Mercury perihelion shift
 $r_0 \approx 0.387$ AU
 $|2\gamma - \beta - 1| < 3 \cdot 10^{-3}$
- Nordtvedt effect
 $r_0 \approx 1$ AU
 $|4\beta - \gamma - 3| < 9 \cdot 10^{-4}$