Parameterized post-Newtonian formalism for scalar-tensor gravity with a potential

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Scalar tensor gravity

- Fields: metric $g_{\mu\nu}$, scalar field Ψ, matter.
- Action:

$$S = S_m[g_{\mu\nu}, \chi_m] + \frac{1}{2\kappa^2}\int_{V_4} d^4x\sqrt{-g}\Big(\Psi R - \frac{\omega(\Psi)}{\Psi}\partial_\rho\Psi\partial^\rho\Psi - 2\kappa^2 V(\Psi)\Big).$$

- Metric field equation:

$$R_{\mu\nu} = \frac{1}{\Psi}\Big[\kappa^2\big(T_{\mu\nu} - \frac{\omega+1}{2\omega+3}g_{\mu\nu}T\big) + \nabla_\mu\partial_\nu\Psi + \frac{\omega}{\Psi}\partial_\mu\Psi\partial_\nu\Psi - \frac{g_{\mu\nu}}{4\omega+6}\frac{d\omega}{d\Psi}\partial_\rho\Psi\partial^\rho\Psi\Big] + \frac{\kappa^2}{\Psi}g_{\mu\nu}\frac{2\omega+1}{2\omega+3}V + g_{\mu\nu}\frac{\kappa^2}{2\omega+3}\frac{dV}{d\Psi}$$

- Scalar field equation:

$$\Box\Psi = \frac{1}{2\omega+3}\Big[\kappa^2 T - \frac{d\omega}{d\Psi}\partial_\rho\Psi\partial^\rho\Psi + 2\kappa^2\big(\Psi\frac{dV}{d\Psi} - 2V\big)\Big]$$

Post-Newtonian approximation

- Expansion around background solution:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,,\ \Psi = \Psi_0 + \psi\,.$$

- Taylor expansion:

$$\omega = \omega_0 + \omega_1\psi + \mathcal{O}(\psi^2)\,,$$
$$V = V_2\psi^2 + V_3\psi^3 + \mathcal{O}(\psi^4)\,.$$

- Mass of the scalar field:

$$m_\psi = 2\kappa\sqrt{\frac{\Psi_0 V_2}{2\omega_0+3}}\,.$$

- Perfect fluid matter with density ρ, internal energy $\rho\Pi$, pressure p, velocity $v^i = \frac{u^i}{u^0}$:

$$T^{\mu\nu} = (\rho + \rho\Pi + p)u^\mu u^\nu + pg^{\mu\nu}\,.$$

- Velocity orders $\mathcal{O}(n) \sim |\vec{v}|^n$:

$$h_{00} = h_{00}^{(2)} + h_{00}^{(4)} + \mathcal{O}(6)\,,$$
$$h_{0i} = h_{0i}^{(3)} + \mathcal{O}(5)\,,\ h_{ij} = h_{ij}^{(2)} + \mathcal{O}(4)\,,$$
$$\psi = \psi^{(2)} + \psi^{(4)} + \mathcal{O}(6)\,.$$

- Gauge conditions:

$$h_i{}^j{}_{,j} - \frac{1}{2}h_\mu{}^\mu{}_{,i} = \frac{1}{\Psi_0}\psi_{,a}\,,$$
$$h_0{}^j{}_{,j} - \frac{1}{2}h_j{}^j{}_{,0} = \frac{1}{\Psi_0}\psi_{,0}\,.$$

Point mass approximation

- Matter source:

$$\rho = M\delta(\vec{x})\,,\ \Pi = 0\,,\ p = 0\,,\ v_i = 0\,.$$

- Newtonian potential:

$$U(r) = \frac{\kappa^2}{8\pi}\frac{M}{r}\,.$$

- Gravitational self-energy:

$$\Phi^{(4)} = 0\,.$$

Newtonian limit

- Scalar field:

$$\psi^{(2)}(r) = \frac{2}{2\omega_0+3}U(r)e^{-m_\psi r}\,.$$

- Metric perturbation:

$$h_{00}^{(2)}(r) = 2G_{\text{eff}}(r)U(r)\,.$$

- Effective Newtonian constant:

$$G_{\text{eff}}(r) = \frac{1}{\Psi_0}\left(1 + \frac{e^{-m_\psi r}}{2\omega_0+3}\right)\,.$$

PPN parameter $\gamma(r)$

- Metric perturbation:

$$h_{ij}^{(2)}(r) = 2G_{\text{eff}}(r)\gamma(r)U(r)\delta_{ij}\,.$$

- Solution for $\gamma(r)$:

$$\gamma(r) = \frac{2\omega_0+3-e^{-m_\psi r}}{2\omega_0+3+e^{-m_\psi r}}\,.$$

- L. Perivolaropoulos 2009.
- G. J. Olmo 2005.
- Y. Xie *et al.* 2007.

PPN parameter $\beta(r)$

- Scalar field:

$$\psi^{(4)}(r) = \frac{2}{(2\omega_0+3)^2}\left(\frac{1}{\Psi_0} - \frac{\omega_1}{2\omega_0+3}\right)U^2(r)e^{-2m_\psi r} + \frac{2m_\psi}{\Psi_0(2\omega_0+3)}U^2(r)r\left(e^{m_\psi r}\,\text{Ei}(-2m_\psi r) - e^{-m_\psi r}\ln(m_\psi r)\right) + \frac{3m_\psi}{(2\omega_0+3)^2}\left(\frac{V_3}{V_2} - \frac{1}{\Psi_0} - \frac{\omega_1}{2\omega_0+3}\right)U^2(r)r\left(e^{m_\psi r}\,\text{Ei}(-3m_\psi r) - e^{-m_\psi r}\,\text{Ei}(-m_\psi r)\right)\,.$$

- Metric perturbation:

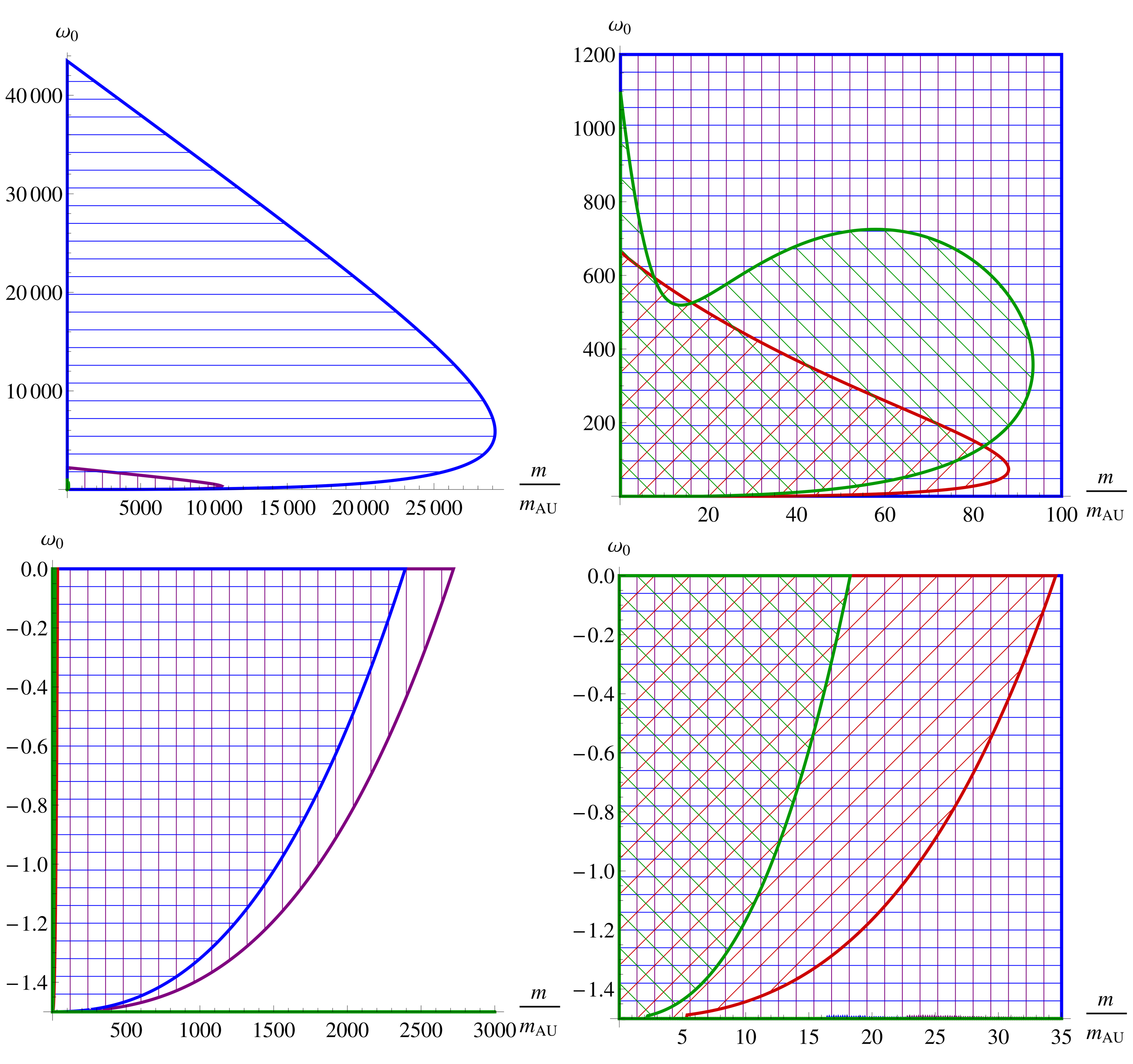
$$h_{00}^{(4)}(r) = -2G_{\text{eff}}^2(r)\beta(r)U^2(r) + \Phi^{(4)}(r)\,.$$

- Potential $\Phi^{(4)}(r)$ absorbs terms $\sim \rho U$ coming from gravitational self-energy.
- Solution for $\beta(r)$:

$$\beta(r) = 1 + \frac{\omega_1 e^{-2m_\psi r}}{G_{\text{eff}}^2(r)\Psi_0(2\omega_0+3)^3} - \frac{m_\psi r}{G_{\text{eff}}^2(r)(2\omega_0+3)}\Bigg[\frac{2}{\Psi_0^2}\left[e^{-m_\psi r} + \text{Ei}(-m_\psi r)m_\psi r\right] + \frac{1}{2\Psi_0(2\omega_0+3)}\left(\frac{2\omega_0-2}{\Psi_0} + \frac{3V_3}{V_2} - \frac{\omega_1}{2\omega_0+3}\right)\left[e^{-2m_\psi r} + 2\,\text{Ei}(-2m_\psi r)m_\psi r\right] + \frac{1}{\Psi_0^2}\left(2e^{-m_\psi r} - e^{m_\psi r}\,\text{Ei}(-2m_\psi r) + 2\,\text{Ei}(-m_\psi r)m_\psi r + e^{-m_\psi r}\ln(m_\psi r)\right) + \frac{3}{2\Psi_0(2\omega_0+3)}\left(\frac{V_3}{V_2} - \frac{1}{\Psi_0} - \frac{\omega_1}{2\omega_0+3}\right) \times \left(-e^{-2m_\psi r} + e^{m_\psi r}\,\text{Ei}(-3m_\psi r) - 2\,\text{Ei}(-2m_\psi r)m_\psi r - e^{-m_\psi r}\,\text{Ei}(-m_\psi r)\right)\Bigg]\,.$$

- MH, LJ, PK, ER, Phys. Rev. D **88** 084054 (2013)

Comparison with experiments: lined regions ruled out



- Effect of ω_1, V_3 negligible.
- x-axis: mass parameter

$$m = 2\kappa\sqrt{\Psi_0 V_2}\,.$$

- y-axis: ω_0.

Very long baseline interferometry
$r_0 \approx 4.65\cdot 10^{-3}\,\text{AU}$
$\gamma - 1 = (-1.7 \pm 4.5)\cdot 10^{-4}$

Cassini tracking
$r_0 \approx 7.44\cdot 10^{-3}\,\text{AU}$
$\gamma - 1 = (2.1 \pm 2.3)\cdot 10^{-5}$

Mercury perihelion shift
$r_0 \approx 0.387\,\text{AU}$
$|2\gamma - \beta - 1| < 3\cdot 10^{-3}$

Nordtvedt effect
$r_0 \approx 1\,\text{AU}$
$|4\beta - \gamma - 3| < 9\cdot 10^{-4}$