Parameterized post-Newtonian limit of Horndeski's gravity theory

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Overview

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  - Scalar field in addition to metric mediating gravity?
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- Horndeski gravity [G. W. Horndeski '74]:
  - Scalar-tensor theory of gravity.
  - Most general STG with second order field equations.
  - Healthy, ghost-free theory.
  - Contains many interesting cases (Galileons, Higgs inflation...).

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  - PPN parameters expressed by invariants [L. Järv, P. Kuusk, M. Saal, O. Vilson '15].
- Horndeski's theory: generalizes aforementioned theories.

Gravitational action

- Action functional [T. Kobayashi, M. Yamaguchi, J. 'i. Yokoyama '11]:

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- Gravitational Lagrangian with $X = -\nabla_\mu\phi\nabla^\mu\phi/2$:

$$\begin{aligned}
\mathcal{L}_2 &= K(\phi, X)\,,\\
\mathcal{L}_3 &= -G_3(\phi, X)\Box\phi\,,\\
\mathcal{L}_4 &= G_4(\phi, X)R + G_{4X}(\phi, X)\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\,,\\
\mathcal{L}_5 &= G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi\\
&\quad - \frac{1}{6}G_{5X}(\phi, X)\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right]\,.
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\end{aligned}$$

- Free functions K, G_3, G_4, G_5.

Field equations

- Structure of the field equations:

$$\sum_{i=2}^{5} \mathcal{G}^{i}_{\mu\nu} = \frac{1}{2} T_{\mu\nu} \,, \quad \sum_{i=2}^{5} \nabla^{\mu} J^{i}_{\mu} = \sum_{i=2}^{5} P^{i}_{\phi} \,.$$

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- More convenient: trace-reversed field equations:

$$\sum_{i=2}^{5} \mathcal{R}^i_{\mu\nu} = \frac{1}{2} \bar{T}_{\mu\nu} = \frac{1}{2}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)\,.$$

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- Geometry tensors:

$$\mathcal{R}^i_{\mu\nu} = \mathcal{G}^i_{\mu\nu} - \frac{1}{2} g_{\mu\nu} g^{\rho\sigma} \mathcal{G}^i_{\rho\sigma}\,.$$

Perturbative expansion

- Background solution:
  - Minkowski metric $\eta_{\mu\nu}$
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- Taylor expansion of free functions:

$$K(\phi, X) = \sum_{m,n=0}^{\infty} K_{(m,n)}\psi^{m}X^{n}\,.$$

- Expansion coefficients:

$$K_{(m,n)} = \frac{1}{m!n!}\,\frac{\partial^{m+n}}{\partial\phi^{m}\partial X^{n}}K(\phi, X)\bigg|_{\phi=\Phi,X=0}\,.$$

- Similar expansion for G_3, G_4, G_5.

Post-Newtonian approximation

- Perfect fluid energy-momentum tensor:

$$T^{\mu\nu} = (\rho + \rho\Pi + p)u^{\mu}u^{\nu} + pg^{\mu\nu}\,.$$

  - Four-velocity u^{μ}.
  - Matter density ρ.
  - Specific internal energy Π.
  - Pressure p.

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  - Matter density $\rho \sim \mathcal{O}(2)$.
  - Specific internal energy $\Pi \sim \mathcal{O}(2)$.
  - Pressure $p \sim \mathcal{O}(4)$.
- Slow-moving source matter:

$$v^i = \frac{u^i}{u^0} \ll 1 \,.$$

- Assign velocity orders $|v^i|^n \sim \mathcal{O}(n)$ based on solar system.

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- Relevant terms for dynamical fields:

$$g_{00} = -1 + h^{(2)}_{00} + h^{(4)}_{00} + \mathcal{O}(6) , \quad g_{0j} = h^{(3)}_{0j} + \mathcal{O}(5) ,$$
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- Time dependence only through motion of source matter.

$\Rightarrow$ Assign time derivative $\partial_0 \sim \mathcal{O}(1)$.

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Spherically symmetric solution

- Static, point-like mass source:

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$$\begin{aligned}
g_{00} &= -1 + 2G_{\text{eff}}(r)U(r) - 2G_{\text{eff}}^2(r)\beta(r)U^2(r) + \Phi^{(4)}(r) + \mathcal{O}(6)\,,\\
g_{0j} &= \mathcal{O}(5)\,,\\
g_{ij} &= [1 + 2G_{\text{eff}}(r)\gamma(r)U(r)]\,\delta_{ij} + \mathcal{O}(4)\,.
\end{aligned}$$

  - Newtonian potential: $U(r) = M/r$.
  - Gravitational self energy $\Phi^{(4)}(r)$.
  - Effective gravitational constant $G_{\text{eff}}(r)$.
  - PPN parameters $\gamma(r)$ and $\beta(r)$.

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- Consistency condition:

$$K_{(0,0)} = K_{(1,0)} = 0\,.$$

Scalar field $\psi^{(2)}$

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- Constants:

$$m_\psi = \sqrt{\frac{-2K_{(2,0)}}{K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}}}}\,,$$

$$c_\psi = \frac{G_{4(1,0)}}{2G_{4(0,0)}}\left(K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}}\right)^{-1}\,.$$

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- Constants:

$$c_1 = -2\frac{G_{4(1,0)}K_{(2,0)}}{G_{4(0,0)}}\left(K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}}\right)^{-1} ,$$

$$c_2 = \frac{1}{G_{4(0,0)}}\left[\frac{1}{2} + \frac{G_{4(1,0)}^2}{2G_{4(0,0)}}\left(K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G_{4(1,0)}^2}{G_{4(0,0)}}\right)^{-1}\right] .$$

PPN parameter γ

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$$\gamma(r) = \frac{c_4 + \frac{c_3 c_\psi}{m_\psi^2}(e^{-m_\psi r} - 1)}{c_2 + \frac{c_1 c_\psi}{m_\psi^2}(e^{-m_\psi r} - 1)}\,.$$

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$$c_3 = 2\frac{G_{4(1,0)}K_{(2,0)}}{G_{4(0,0)}}\left(K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}}\right)^{-1}\,,$$

$$c_4 = \frac{1}{G_{4(0,0)}}\left[\frac{1}{2} - \frac{G^2_{4(1,0)}}{2G_{4(0,0)}}\left(K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}}\right)^{-1}\right]\,.$$

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$$h^{(2)}_{ij,kk} = \left(c_3\psi^{(2)} - c_4\rho\right)\delta_{ij}\,.$$

- Solve and read off PPN parameter γ:

$$\gamma(r) = \frac{2\omega + 3 - e^{-m_\psi r}}{2\omega + 3 + e^{-m_\psi r}}\,.$$

- Constants:

$$\omega = \frac{G_{4(0,0)}}{2G^2_{4(1,0)}}\left(K_{(0,1)} - 2G_{3(1,0)}\right)\,,$$

$$m_\psi = \sqrt{\frac{-2K_{(2,0)}}{K_{(0,1)} - 2G_{3(1,0)} + 3\frac{G^2_{4(1,0)}}{G_{4(0,0)}}}}\,.$$

PPN parameter β

- Calculate β from fourth order solution:

$$
\begin{aligned}
\beta(r) = 1 + \frac{1}{(2\omega + 3 + e^{-m_\psi r})^2} \Bigg\{ &\frac{\omega + \tau - 4\omega\sigma}{2\omega + 3} e^{-2m_\psi r} \\
&+ (2\omega + 3) m_\psi r \Bigg[e^{-m_\psi r} \ln(m_\psi r) - \frac{1}{2} e^{-2m_\psi r} \\
&\quad - (m_\psi r + e^{m_\psi r}) \operatorname{Ei}(-2m_\psi r) \Bigg] \\
&+ \frac{6\mu r + 3(3\omega + \tau + 6\sigma + 3) {m_\psi}^2 r}{2(2\omega + 3) m_\psi} \big[e^{m_\psi r} \operatorname{Ei}(-3m_\psi r) \\
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\end{aligned}
$$

- Constants $m_\psi, \omega, \tau, \sigma, \mu$.

Limiting cases

- $m_\psi \to 0$, all other constants fixed and finite:

$$\gamma = \frac{\omega + 1}{\omega + 2}\,, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{(2\omega + 3)(2\omega + 4)^2}\,.$$

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- $m_\psi r \to \infty$, large distance from the matter source:

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Full PPN parameters for massless theory

- Consider more restricted theory:

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$\Rightarrow$ PPN limit assumes standard form with constant PPN parameters.

- PPN parameters:

$$\gamma = \frac{\omega + 1}{\omega + 2}\,, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{4(\omega + 2)^2(2\omega + 3)}\,,$$
$$\alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = \xi = 0\,.$$

$\Rightarrow$ Only γ and β potentially deviate from observed values.

Overview

Large mass limit

- Asymptotic behavior of exponential integral:

$$\mathrm{Ei}(-x) \approx \frac{e^{-x}}{x}\left(1 - \frac{1!}{x} + \frac{2!}{x^2} - \frac{3!}{x^3} + \dots\right)\,.$$

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$\Rightarrow$ Terms involving σ, τ, $\mu \sim e^{-2m_\psi r}$ are subleading.
$\Rightarrow$ Consider simplified PPN parameters for $m_\psi r \gg 1$:

$$\gamma(r) = 1 - \frac{2}{2\omega + 3} e^{-m_\psi r} + \mathcal{O}(e^{-2m_\psi r}),$$
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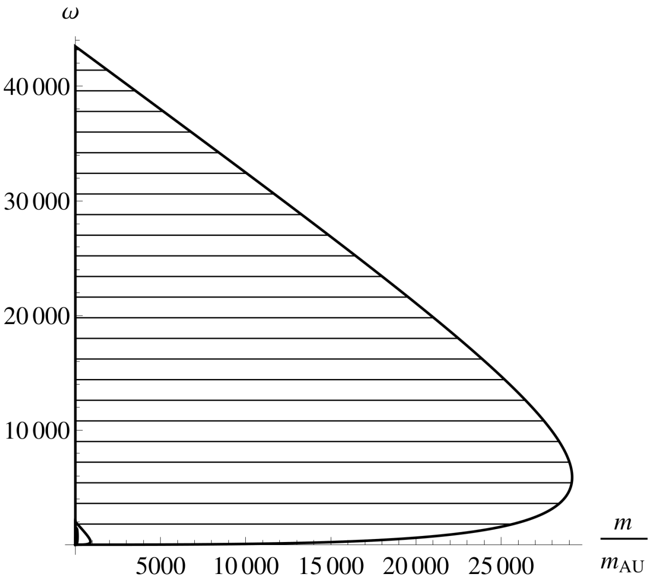
⇒ Need experiments with fixed interaction distance r.

- Most stringent bounds from Cassini tracking [B. Bertotti, L. Iess, P. Tortora '03]:

$$\gamma - 1 = (2.1 \pm 2.3) \cdot 10^{-5} \quad \text{at} \quad r \approx 7.44 \cdot 10^{-3}\,\mathrm{AU}.$$

Excluded parameter ranges at 2σ



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$\Rightarrow$ Possible to use observations where r is not well-defined.

- INPOP13 ephemeris [A. Fienga, P. Exertier, M. Gastineau, J. Laskar, H. Manche, A. Verma '13/'14]:

$$\gamma - 1 = (-0.3 \pm 2.5) \cdot 10^{-5}\,, \quad \beta - 1 = (0.2 \pm 2.5) \cdot 10^{-5}\,.$$

Small mass limit

- PPN parameters independent of r for $m_\psi r \ll 1$:

$$\gamma = \frac{\omega + 1}{\omega + 2}\,, \quad \beta = 1 + \frac{\omega + \tau - 4\omega\sigma}{4(\omega + 2)^2(2\omega + 3)}\,.$$

$\Rightarrow$ Possible to use observations where r is not well-defined.

- INPOP13 ephemeris [A. Fienga, P. Exertier, M. Gastineau, J. Laskar, H. Manche, A. Verma '13/'14]:

$$\gamma - 1 = (-0.3 \pm 2.5) \cdot 10^{-5}\,, \quad \beta - 1 = (0.2 \pm 2.5) \cdot 10^{-5}\,.$$

- Still more stringent bounds by including Cassini tracking:

$$-2.5 \cdot 10^{10} \leq \tau - 4\omega\sigma \leq 2.7 \cdot 10^{10} \quad \text{for} \quad \omega = 4.0 \cdot 10^4\,.$$

- Less stringent bounds for larger values of ω.

How to get better bounds?

- Stringency of bounds depends on interaction distance r_0.
  - Experiments with smaller r_0 are more sensitive to $m_\psi > 0$.
  - Measure γ and β at shorter distances.
  - Earth-moon system? Satellite missions?

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  - Earth-moon system? Satellite missions?
- Can lunar laser ranging help?
  - Nordvedt effect depends on γ and β.
  - But: Nordvedt effect concerns motion in solar gravitational field.
  - Interaction distance $r_0 = 1$AU is large.
  - Not the kind of experiment we need.

Overview

Scalar-tensor gravity with potential

- Gravitational action:

$$S_G = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(\phi R - \frac{\omega(\phi)}{\phi}\partial_\rho\phi\partial^\rho\phi - 2\kappa^2 V(\phi)\right) .$$

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- PPN parameters [MH, L. Järv, P. Kuusk, E. Randla '13]:

$$\gamma(r) = \frac{2\omega_0 + 3 - e^{-m_\psi r}}{2\omega_0 + 3 + e^{-m_\psi r}} ,$$

$$\begin{aligned}\beta(r) = 1 + \frac{1}{(2\omega_0 + 3 + e^{-m_\psi r})^2}\Bigg\{&\frac{\Phi\omega_1}{2\omega_0 + 3}e^{-2m_\psi r} + (2\omega_0 + 3)m_\psi r\\ &\times\left[e^{-m_\psi r}\ln(m_\psi r) - (m_\psi r + e^{m_\psi r})\,\mathrm{Ei}(-2m_\psi r) - \frac{1}{2}e^{-2m_\psi r}\right]\\ &+\frac{3m_\psi r}{2}\left(1 - \frac{\Phi V_3}{V_2} + \frac{\Phi\omega_1}{2\omega_0 + 3}\right)\left[e^{m_\psi r}\mathrm{Ei}(-3m_\psi r) - e^{-m_\psi r}\mathrm{Ei}(-m_\psi r)\right]\Bigg\} .\end{aligned}$$

Non-minimal Higgs inflation

- Gravitational action [F. L. Bezrukov, M. Shaposhnikov '08]:

$$S_G = \int d^4x \sqrt{-g} \left(\frac{M_{\mathrm{Pl}}^2 - \xi\phi^2}{2} R + X - V(\phi) \right) .$$

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- Higgs field: $m_\psi = 125\mathrm{GeV}$, $\Phi = 246\mathrm{GeV}$.

$\Rightarrow$ $\gamma = \beta = 1$ on any astrophysical scale.

Overview

Summary

- Horndeski's gravity theory:
  - Most general scalar-tensor theory with second order equations.
  - Four free functions of ϕ and $X = -\nabla^\mu\phi\nabla_\mu\phi/2$.

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- Example theories:
  - Classical scalar-tensor gravity with arbitrary potential.
  - Models of Higgs inflation.
  - Galileons.
- PPN parameters:
  - Most general theory: obtained PPN parameters $\gamma(r)$ and $\beta(r)$.
  - Massless scalar field: only γ and β potentially deviate.
  - Reproduces and generalizes well-known results.
  - Many example theories compatible with solar system observations.

Outlook

- Extend analysis to more general theories:
  - Allow time-dependent scalar background field $\dot{\Phi} \neq 0$.
  - Theories beyond Horndeski / G^3-inflation.
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Outlook

- Extend analysis to more general theories:
  - Allow time-dependent scalar background field $\dot{\Phi} \neq 0$.
  - Theories beyond Horndeski / G^3-inflation.
  - Multi-scalar Horndeski gravity.
- Take screening mechanisms into account:
  - Vainshtein mechanism.
  - Chameleon mechanism.
  - Symmetron mechanism.