Gravitational waves in teleparallel gravity

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European Union
European Regional
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Athens - 25. January 2019

Outline

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- Open questions in cosmology and gravity:
  - Accelerating phases in the history of the Universe?
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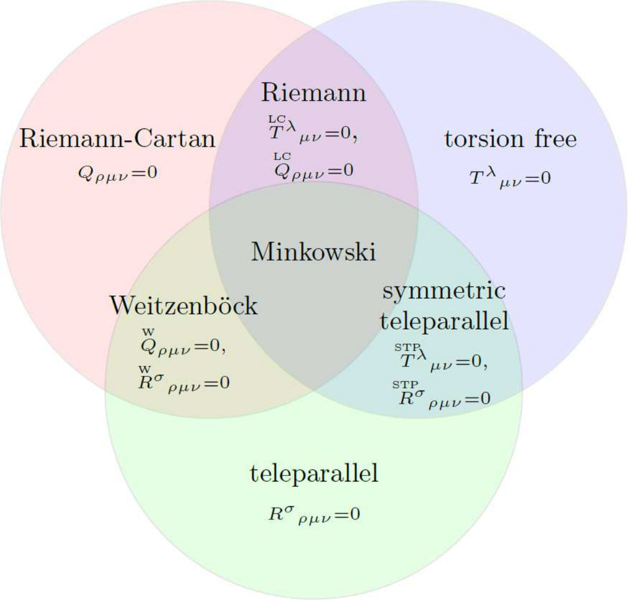
- Gravity formulated as gauge theories.

Overview of geometries



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- Consider plane wave ansatz $\Psi^A(x) = \hat{\Psi}^A e^{ik_\mu x^\mu}$ for the field:

$$D^A{}_B\Psi^B(x) = \left(M^A{}_B(x) + \ldots + i^p M^A{}_B{}^{\mu_1\cdots\mu_m}(x)k_{\mu_1}\cdots k_{\mu_m}\right)\hat{\Psi}^A e^{ik_\mu x^\mu}\,.$$

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- Principal symbol is the highest order term in wave covector k_μ:

$$P^A{}_B(x,k) = M^A{}_B{}^{\mu_1\cdots\mu_m}(x)k_{\mu_1}\cdots k_{\mu_m}\,.$$

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  - Foliation of spacetime by spacelike hypersurfaces with covector $\tilde{k}_\mu$.
  - Initial data on chosen hypersurface $t = 0$.
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⇒ Propagation speed determined by zeros of principal polynomial.

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Newman-Penrose formalism

- Complex double null basis of the tangent bundle:

$$l = \partial_t + \partial_z \,, \quad n = \frac{\partial_t - \partial_z}{2} \,, \quad m = \frac{\partial_x + i\partial_y}{\sqrt{2}} \,, \quad \bar{m} = \frac{\partial_x - i\partial_y}{\sqrt{2}} \,.$$

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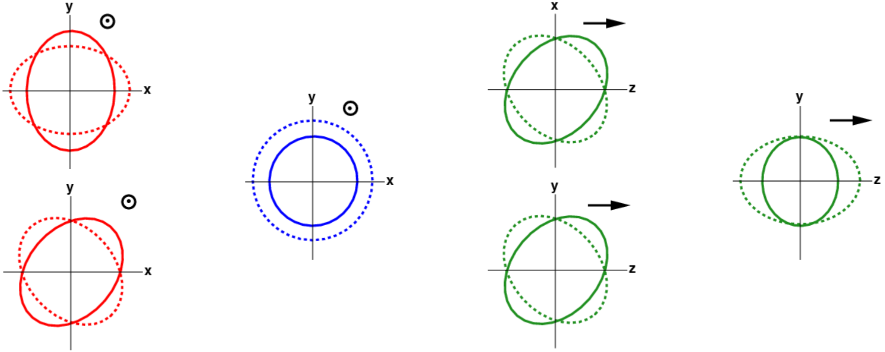
- Riemann tensor determined by “electric” components:

$$\Psi_2 = -\frac{1}{6} R_{nlnl} = \frac{1}{12}\ddot{h}_{ll}\,, \qquad \Psi_3 = -\frac{1}{2} R_{nln\bar{m}} = \frac{1}{4}\ddot{h}_{l\bar{m}}\,,$$

$$\Psi_4 = -R_{n\bar{m}n\bar{m}} = \frac{1}{2}\ddot{h}_{\bar{m}\bar{m}}\,, \qquad \Phi_{22} = -R_{nmn\bar{m}} = \frac{1}{2}\ddot{h}_{m\bar{m}}\,.$$

Polarisations of gravitational waves

Effect of the different polarizations on spherical shell of test masses:



$\Psi_4, \bar{\Psi}_4$	Φ_{22}	$\Psi_3, \bar{\Psi}_3$	Ψ_2
tensors	breathing	vectors	longitudinal

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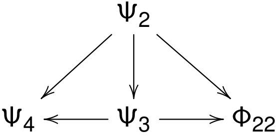

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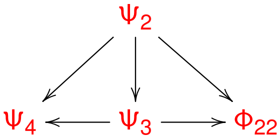

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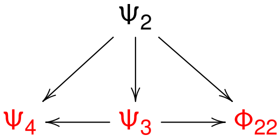

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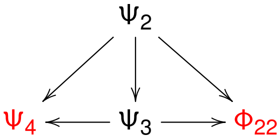

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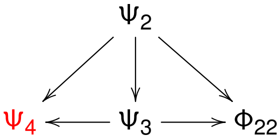

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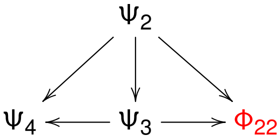

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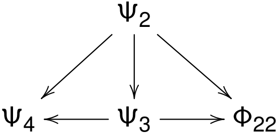

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  - O_0: no gravitational waves.

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  - Levi-Civita connection

$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})\,.$$

  - Non-metricity $Q_{\rho\mu\nu} = \overset{\times}{\nabla}_\rho g_{\mu\nu}$.

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$$\overset{\circ}{\Gamma}{}^{\rho}{}_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu})\,.$$

  - Non-metricity $Q_{\rho\mu\nu} = \overset{\times}{\nabla}{}_{\rho} g_{\mu\nu}$.
- Gauge fixing
  - Perform local coordinate transformation:

$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}g_{\alpha\beta}\,, \quad \overset{\times}{\Gamma}{}'^{\rho}{}_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu}\frac{\partial x^\beta}{\partial x'^\nu}\frac{\partial x'^\rho}{\partial x^\gamma}\overset{\times}{\Gamma}{}^{\gamma}{}_{\alpha\beta} + \frac{\partial^2 x^\alpha}{\partial x'^\mu \partial x'^\nu}\frac{\partial x'^\rho}{\partial x^\alpha}\,.$$

$\Rightarrow$ Coincident gauge: set $\overset{\times}{\Gamma}{}^{\rho}{}_{\mu\nu} \equiv 0 \Rightarrow Q_{\rho\mu\nu} = \partial_\rho g_{\mu\nu}$.

Most general action and corresponding field equations

- Most general action:

$$S = -\int d^4x \frac{\sqrt{-g}}{2}\Big[c_1 Q^{\alpha}{}_{\mu\nu} + c_2 Q_{(\mu}{}^{\alpha}{}_{\nu)} + c_3 Q^{\alpha} g_{\mu\nu} + c_4 \delta^{\alpha}_{(\mu}\tilde{Q}_{\nu)} + \frac{c_5}{2}\left(\tilde{Q}^{\alpha} g_{\mu\nu} + \delta^{\alpha}_{(\mu} Q_{\nu)}\right)\Big] Q_{\alpha}{}^{\mu\nu}\,.$$

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,.$$

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$$\begin{aligned}0 = {} & 2c_1\eta^{\alpha\sigma}\partial_\alpha\partial_\sigma h_{\mu\nu} + c_2\eta^{\alpha\sigma}\left(\partial_\alpha\partial_\mu h_{\sigma\nu} + \partial_\alpha\partial_\nu h_{\sigma\mu}\right) \\ & + 2c_3\eta_{\mu\nu}\eta^{\tau\omega}\eta^{\alpha\sigma}\partial_\alpha\partial_\sigma h_{\tau\omega} + c_4\eta^{\omega\sigma}(\partial_\mu\partial_\omega h_{\nu\sigma} + \partial_\nu\partial_\omega h_{\mu\sigma}) \\ & + c_5\eta_{\mu\nu}\eta^{\omega\gamma}\eta^{\alpha\sigma}\partial_\alpha\partial_\omega h_{\sigma\gamma} + c_5\eta^{\omega\sigma}\partial_\mu\partial_\nu h_{\omega\sigma}\,.\end{aligned}$$

Principal polynomial and speed of propagation

- Decomposition of amplitude $\hat{h}_{\lambda\rho}$ in irreducible components:

$$\hat{h}_{\lambda\rho} = S_{\lambda\rho} + 2k_{(\lambda}V_{\rho)} + \frac{1}{3}\left(\eta_{\lambda\rho} - \frac{k_\lambda k_\rho}{\eta^{\mu\nu}k_{\mu\nu}}\right)T + \left(k_\lambda k_\rho - \frac{1}{4}\eta_{\lambda\rho}\eta^{\alpha\beta}k_\alpha k_\beta\right)U\,.$$

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$$0 = (2c_3 + c_5)(\eta^{\alpha\beta}k_\alpha k_\beta)^2 T + \frac{3}{4}[c_5 + 2(c_1 + c_2 + c_4)](\eta^{\alpha\beta}k_\alpha k_\beta)^3 U\,,$$

$$0 = (2c_1 + 8c_3 + c_5)(\eta^{\alpha\beta}k_\alpha k_\beta)T + \frac{3}{2}(2c_5 + c_2 + c_4)(\eta^{\alpha\beta}k_\alpha k_\beta)^2 U\,,$$

$$0 = (2c_1 + c_2 + c_4)(\eta^{\alpha\beta}k_\alpha k_\beta)^2 V_\nu\,,$$

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- $\eta^{\alpha\beta}k_\alpha k_\beta = 0 \Leftrightarrow$ propagation at the speed of light.

Newman-Penrose formalism

- Assume plane null wave $h_{\mu\nu} = \hat{h}_{\mu\nu} e^{ik_\mu x^\mu}$ with $\eta^{\alpha\beta} k_\alpha k_\beta = 0$.

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$\Rightarrow$ Field equations expressed in Newman-Penrose basis:

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0 &= E_{nn} = -(c_2 + c_4 + c_5)\ddot{h}_{ln} + c_5 \ddot{h}_{m\bar{m}}\,,\\
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  - $c_2 + c_4 \neq 0$, $c_2 + c_4 + c_5 \neq 0$: also vector $\Psi_3 \sim \ddot{h}_{lm}$ prohibited $\Rightarrow$ N_3.

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0 &= E_{nn} = -(c_2 + c_4 + c_5)\ddot{h}_{ln} + c_5 \ddot{h}_{m\bar{m}}\,, \\
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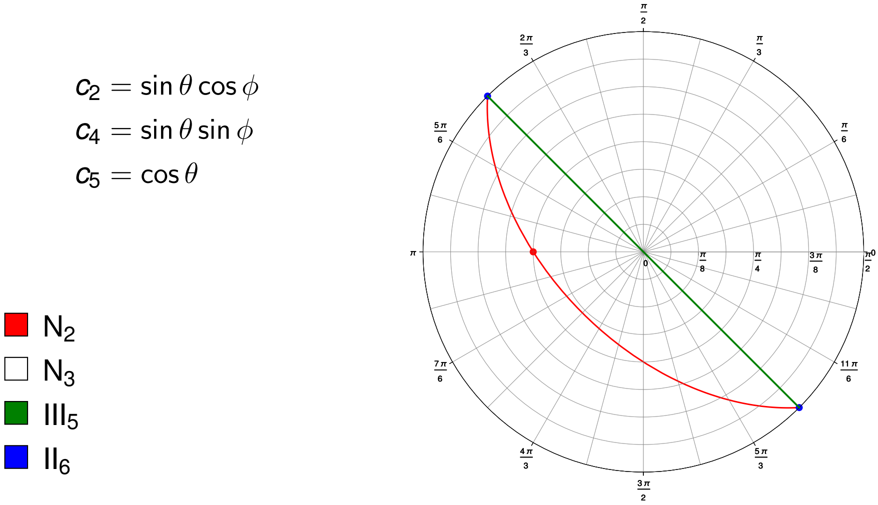
- Possible E(2) classes:
  - (blue) $c_2 + c_4 = c_5 = 0$: all six modes are allowed ⇒ II_6.
  - (green) $c_2 + c_4 = 0$, $c_5 \neq 0$: only scalar $\Psi_2 \sim \ddot{h}_{ll}$ prohibited ⇒ III_5.
  - (white) $c_2 + c_4 \neq 0$, $c_2 + c_4 + c_5 \neq 0$: also vector $\Psi_3 \sim \ddot{h}_{lm}$ prohibited ⇒ N_3.
  - (red) $c_2 + c_4 + c_5 = 0$, $c_5 \neq 0$: also scalar $\Phi_{22} \sim \ddot{h}_{m\bar{m}}$ prohibited ⇒ N_2.

Gravitational wave polarisations

$$c_2 = \sin\theta\cos\phi$$
$$c_4 = \sin\theta\sin\phi$$
$$c_5 = \cos\theta$$



Outline

Field content and geometry

- Fundamental fields in the gravity sector:
  - Coframe field $\theta^a = \theta^a{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\overset{\bullet}{\omega}{}^a{}_b = \overset{\bullet}{\omega}{}^a{}_{b\mu} \mathrm{d}x^\mu$.

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- Derived quantities:
  - Frame field $e_a = e_a{}^\mu \partial_\mu$ with $\iota_{e_a}\theta^b = \delta^b_a$.
  - Metric $g_{\mu\nu} = \eta_{ab}\theta^a{}_\mu \theta^b{}_\nu$.
  - Volume form $\theta \mathrm{d}^4x = \theta^0 \wedge \theta^1 \wedge \theta^2 \wedge \theta^3$.
  - Levi-Civita connection

$$\overset{\circ}{\omega}_{ab} = -\frac{1}{2}(\iota_{e_b}\iota_{e_c}\mathrm{d}\theta_a + \iota_{e_c}\iota_{e_a}\mathrm{d}\theta_b - \iota_{e_a}\iota_{e_b}\mathrm{d}\theta_c)\theta^c \,.$$

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$$\theta'^a = \Lambda^a{}_b\theta^b\,, \quad \overset{\bullet}{\omega}{}'^a{}_b = \Lambda^a{}_c \overset{\bullet}{\omega}{}^c{}_d \Lambda_b{}^d + \Lambda^a{}_c \mathrm{d}\Lambda_b{}^c\,.$$

$\Rightarrow$ Weitzenböck gauge: set $\overset{\bullet}{\omega}{}^a{}_b \equiv 0$.

Most general action and corresponding field equations

- Most general action:

$$S = \frac{1}{2\kappa^2} \int d^4x\, e\,(c_1 T^{\mu\nu\rho} T_{\mu\nu\rho} + c_2 T^{\mu\nu\rho} T_{\rho\nu\mu} + c_3 T^{\mu}{}_{\mu\rho} T_{\nu}{}^{\nu\rho}) \,.$$

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  - Antisymmetric perturbation part, $a_{\mu\nu} = \tau_{[\mu\nu]}$:

$$B^{\mu\rho\sigma} = (2c_1 - c_2) \left(\partial^\sigma a^{\mu\rho} - \partial^\rho a^{\mu\sigma} \right) + (2c_2 + c_3) \partial^\mu a^{\sigma\rho} .$$

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$$0 = (2c_1 + c_2 + c_3)(\eta^{\alpha\beta}k_\alpha k_\beta)^2 W^\kappa \,, \quad 0 = (2c_1 - c_2)\eta^{\alpha\beta}k_\alpha k_\beta A^{\tau\kappa} \,,$$
$$0 = (2c_1 + c_2 + 3c_3)\eta^{\alpha\beta}k_\alpha k_\beta Q^\tau{}_\tau \,, \quad 0 = (2c_1 + c_2)\eta^{\alpha\beta}k_\alpha k_\beta S^{\tau\kappa} \,.$$

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- Write $Q_{\alpha\beta}$ in trace, symmetric traceless and antisymmetric part:

$$Q^{\tau\kappa} = S^{\tau\kappa} + A^{\tau\kappa} + \frac{1}{3}\left(\eta^{\tau\kappa} - \frac{k^\tau k^\kappa}{\eta^{\mu\nu}k_\mu k_\nu}\right) Q^\sigma{}_\sigma\,.$$

- Decomposed field equations:

$$0 = (2c_1 + c_2 + c_3)(\eta^{\alpha\beta}k_\alpha k_\beta)^2 W^\kappa\,, \quad 0 = (2c_1 - c_2)\eta^{\alpha\beta}k_\alpha k_\beta A^{\tau\kappa}\,,$$
$$0 = (2c_1 + c_2 + 3c_3)\eta^{\alpha\beta}k_\alpha k_\beta Q^\tau{}_\tau\,, \quad 0 = (2c_1 + c_2)\eta^{\alpha\beta}k_\alpha k_\beta S^{\tau\kappa}\,.$$

- Principal polynomial $\bar{p}(x,k) = \text{const.} \cdot (\eta^{\alpha\beta}k_\alpha k_\beta)^{15}$.

Principal polynomial and speed of propagation

- Decomposition of amplitude $\hat{\tau}_{\lambda\rho}$ relative to wave vector:

$$\hat{\tau}_{\beta\sigma} = k_\beta k_\sigma U + V_\beta k_\sigma + k_\beta W_\sigma + Q_{\beta\sigma}\,.$$

- Conditions imposed on projected components:

$$k_\alpha V^\alpha = 0\,, \quad k_\alpha W^\alpha = 0\,, \quad k_\alpha Q^\alpha{}_\beta = 0\,, \quad k_\alpha Q_\beta{}^\alpha = 0\,.$$

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- Principal polynomial $\bar{p}(x,k) = \text{const.} \cdot (\eta^{\alpha\beta}k_\alpha k_\beta)^{15}$.
- $\eta^{\alpha\beta}k_\alpha k_\beta = 0 \Leftrightarrow$ propagation at the speed of light.

Newman-Penrose formalism

- Assume plane null wave $\tau_{\mu\nu} = \hat{\tau}_{\mu\nu} e^{ik_\mu x^\mu}$ with $\eta^{\alpha\beta} k_\alpha k_\beta = 0$.

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\begin{aligned}
0 &= E_{nn} = (2c_1 + c_2 + c_3)\ddot{\phi}_{nl} + 2c_3\ddot{\phi}_{m\bar{m}} + (2c_1 + c_2 + c_3)\ddot{a}_{nl}\,,\\
0 &= E_{mn} = (2c_1 + c_2)\ddot{\phi}_{ml} + (2c_1 - c_2)\ddot{a}_{ml}\,,\\
0 &= E_{nm} = -c_3\ddot{\phi}_{lm} - (2c_2 + c_3)\ddot{a}_{lm}\,,\\
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  - $2c_1 + c_2 = c_3 = 0$: all six modes are allowed $\Rightarrow \mathrm{II}_6$.

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  - $2c_1(c_2 + c_3) + c_2^2 = 0$, $2c_1 + c_2 + c_3 \neq 0$: only scalar $\Psi_2 \sim \ddot{h}_{ll} = 0$ $\Rightarrow$ III_5.

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- Possible E(2) classes:
  - ■ (blue) $2c_1 + c_2 = c_3 = 0$: all six modes are allowed $\Rightarrow \mathrm{II}_6$.
  - ■ (green) $2c_1(c_2 + c_3) + c_2^2 = 0$, $2c_1 + c_2 + c_3 \neq 0$: only scalar $\Psi_2 \sim \ddot{h}_{ll} = 0$ $\Rightarrow \mathrm{III}_5$.
  - □ $2c_1(c_2 + c_3) + c_2^2 \neq 0$, $2c_1 + c_2 + c_3 \neq 0$: also vector $\Psi_3 \sim \ddot{h}_{lm} = 0$ $\Rightarrow \mathrm{N}_3$.

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$\Rightarrow$ Field equations expressed in Newman-Penrose basis:

$$0 = E_{nn} = (2c_1 + c_2 + c_3)\ddot{\phi}_{nl} + 2c_3\ddot{\phi}_{m\bar{m}} + (2c_1 + c_2 + c_3)\ddot{a}_{nl}\,,$$

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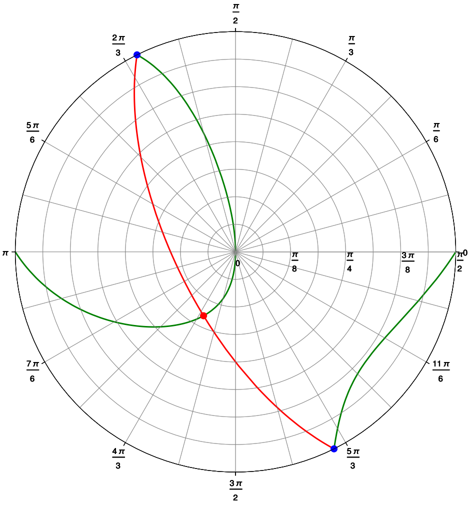
- Possible E(2) classes:
  - (blue) $2c_1 + c_2 = c_3 = 0$: all six modes are allowed $\Rightarrow$ II_6.
  - (green) $2c_1(c_2 + c_3) + c_2^2 = 0$, $2c_1 + c_2 + c_3 \neq 0$: only scalar $\Psi_2 \sim \ddot{h}_{ll} = 0$ $\Rightarrow$ III_5.
  - (white) $2c_1(c_2 + c_3) + c_2^2 \neq 0$, $2c_1 + c_2 + c_3 \neq 0$: also vector $\Psi_3 \sim \ddot{h}_{lm} = 0$ $\Rightarrow$ N_3.
  - (red) $2c_1 + c_2 + c_3 = 0$, $c_3 \neq 0$: also scalar $\Phi_{22} \sim \ddot{h}_{m\bar{m}} = 0 \Rightarrow \mathrm{N}_2$.

$$c_1 = \sin\theta\cos\phi$$
$$c_2 = \sin\theta\sin\phi$$
$$c_3 = \cos\theta$$

- N_2
- N_3
- III_5
- II_6



Outline

Summary

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  - Fields are tetrad and flat spin connection.
  - Only torsion, no curvature or non-metricity.
  - Most general theory needs 3 parameters at linearized level.

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  - Fields are metric and flat, symmetric affine connection.
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  - Most general theory needs 5 parameters at linearized level.
- Results:
  - Gravitational waves propagate at the speed of light.
  - Polarisation classes N_2, N_3, III_5, II_6: tensor + maybe more.

References

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Acknowledgments

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- Estonian Research Council:

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- European Regional Development Fund:

European Union
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- COST Actions CA15117 & CA16104, supported by COST:

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