Solar system consistency of teleparallel gravity

Post-Newtonian limit and PPN parameters of various models

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Outline

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Motivation

- Open questions in gravity theory and cosmology:
  - Accelerating expansion of the universe.
  - Homogeneity of cosmic microwave background.
  - Unification with quantum theory and other fundamental forces.

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  - Modification of gravitational dynamics.
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⇝ Theories considered in this talk:

1. Generic $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ teleparallel gravity.
2. Scalar-torsion analogue and extension of scalar-curvature gravity.
3. Generic $L(X, T, Y, \phi)$ scalar-torsion gravity.

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Post-Newtonian matter and velocity orders

- Energy-momentum tensor of a perfect fluid:

$$\Theta^{\mu\nu} = (\rho + \rho\Pi + p)u^{\mu}u^{\nu} + pg^{\mu\nu}\,.$$

  - Rest mass density ρ.
  - Specific internal energy Π.
  - Pressure p.
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- Universe rest frame and slow-moving source matter:
  - Consider particular coordinates ("universe rest frame").
  - Velocity of the source matter: $v^i = u^i/u^0$.
  - Assume that source matter is slow-moving: $|\vec{v}| \ll 1$.

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- Quasi-static: assign additional $\mathcal{O}(1)$ to time derivatives ∂_0.

Post-Newtonian metric

- Standard post-Newtonian metric expansion:

$$g_{\mu\nu} = \overset{0}{g}_{\mu\nu} + \overset{1}{g}_{\mu\nu} + \overset{2}{g}_{\mu\nu} + \overset{3}{g}_{\mu\nu} + \overset{4}{g}_{\mu\nu} + \mathcal{O}(5)\,.$$

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  - Second-order spatial part $\overset{2}{g}_{ij}$ is diagonal.
  - Fourth-order temporal part $\overset{4}{g}_{00}$ does not contain potential $\mathfrak{B}$.

PPN parameters and potentials

- Metric in standard PPN gauge:

$$\overset{2}{g}_{00} = 2U\,,$$
$$\overset{2}{g}_{ij} = 2\gamma U\delta_{ij}\,,$$
$$\overset{3}{g}_{0i} = -\frac{1}{2}(3 + 4\gamma + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i\,,$$
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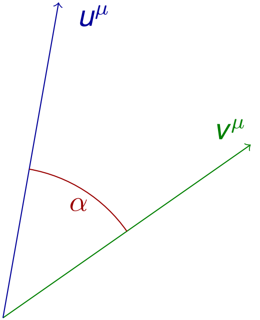
$\rightsquigarrow$ Compare with PPN metric to get PPN parameters.

Outline

General relativity and curvature

Metrics $g_{\mu\nu}$:
$g_{\mu\nu} u^\mu v^\nu = \|u\| \|v\| \cos\alpha$.

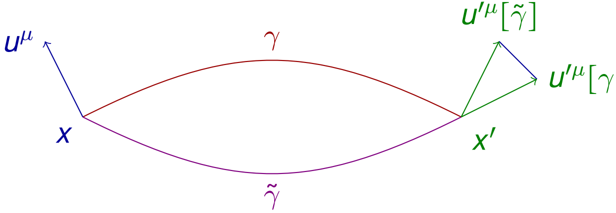
- Vectors u^μ, v^ν.
- Lengths $\|u\|, \|v\|$.
- Angle α.



Connection and parallel transport ∇:

- Moves vector u^μ from x to x'.
- Motion follows trajectory γ.
- Result depends on the trajectory.
- Curvature: difference between results.
- Curvature mediates gravity:

$$S = \int_M \sqrt{-g} R \, d^4x \,.$$



Parallel transport and teleparallel geometry

Tetrad describes geometry:

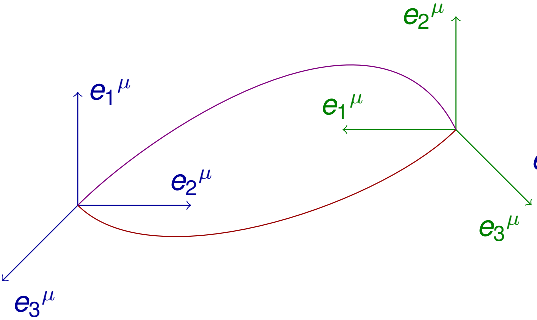
- Frame ${e_a}^\mu$ at every point.
- Frames generate transport.
- Independent of trajectory.

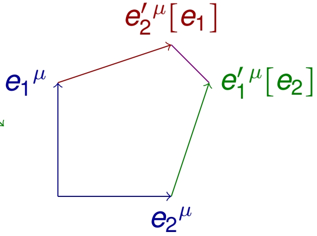
$\Rightarrow$ There is no curvature.

Torsion instead of curvature:

- Non-symmetric transport.
- Torsion $\leftrightsquigarrow$ gravity.
- Gravitational action:

$$S = \frac{1}{2}\int_M H \wedge T\,.$$





Teleparallel geometry

- Fundamental fields:
  - Coframe field $\theta^A = \theta^A{}_\mu \mathrm{d}x^\mu$.
  - Flat spin connection $\omega^A{}_B = \omega^A{}_{B\mu} \mathrm{d}x^\mu$.
  - Arbitrary matter fields χ.

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- Derived quantities:
  - Frame field $e_A = e_A{}^\mu \partial_\mu$ with $e_A{}^\mu \theta^B{}_\mu = \delta^B_A$ and $e_A{}^\mu \theta^A{}_\nu = \delta^\mu{}_\nu$.
  - Metric $g_{\mu\nu} = \eta_{AB} \theta^A{}_\mu \theta^B{}_\nu$.
  - Determinant $\theta = \det(\theta^A{}_\mu)$.
  - Teleparallel connection $\Gamma^\mu{}_{\nu\rho} = e_A{}^\mu (\partial_\rho \theta^A{}_\nu + \omega^A{}_{B\rho} \theta^B{}_\nu)$.
  - Levi-Civita connection $\overset{\circ}{\Gamma}{}^\mu{}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma} (\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho})$.

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- Properties of the teleparallel connection:
  - Vanishing curvature:
    $R^\mu{}_{\nu\rho\sigma} = \partial_\rho \Gamma^\mu{}_{\nu\sigma} - \partial_\sigma \Gamma^\mu{}_{\nu\rho} + \Gamma^\mu{}_{\tau\rho} \Gamma^\tau{}_{\nu\sigma} - \Gamma^\mu{}_{\tau\sigma} \Gamma^\tau{}_{\nu\rho} = 0$.
  - Vanishing nonmetricity: $Q_{\mu\nu\rho} = \nabla_\mu g_{\nu\rho} = 0$.
  - Nonvanishing torsion: $T^\mu{}_{\nu\rho} = \Gamma^\mu{}_{\rho\nu} - \Gamma^\mu{}_{\nu\rho} \neq 0$.

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$\Rightarrow$ Possible to use Weitzenböck gauge: $\omega^A{}_{B\mu} \equiv 0$.

Teleparallel equivalent of general relativity

- Torsion scalar:

$$T = \frac{1}{4} T^{\mu\nu\rho} T_{\mu\nu\rho} + \frac{1}{2} T^{\mu\nu\rho} T_{\rho\nu\mu} - T^{\mu}{}_{\mu\rho} T_{\nu}{}^{\nu\rho} \, .$$

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- Relation between Levi-Civita Ricci scalar and torsion scalar:

$$\overset{\circ}{R} = -T + B = -T + 2\overset{\circ}{\nabla}_{\nu} T_{\mu}{}^{\mu\nu}\,.$$

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- Boundary term does not contribute to field equations.

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- Relation to metric components:

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$$\overset{4}{g}{}_{00} = -(\overset{2}{\theta}{}_{00})^2 + 2\overset{4}{\theta}{}_{00}\,, \quad \overset{4}{g}{}_{ij} = 2\overset{4}{\theta}{}_{(ij)} + \overset{2}{\theta}{}_{ki}\overset{2}{\theta}{}_{kj}\,.$$

Outline

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Action and field equations

- Gravitational part of the action:

$$S_g[\theta, \omega] = \frac{1}{2\kappa^2} \int_M \mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)\, \theta\, \mathrm{d}^4 x\,.$$

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$$\begin{aligned}\kappa^2 \Theta_{\mu\nu} = {}& \frac{1}{2}\mathcal{F} g_{\mu\nu} + 2\overset{\circ}{\nabla}{}^{\rho}\left(\mathcal{F}_{,1} T_{\nu\mu\rho} + \mathcal{F}_{,2} T_{[\rho\mu]\nu} + \mathcal{F}_{,3} T^{\sigma}{}_{\sigma[\rho} g_{\mu]\nu}\right) \\ & + \mathcal{F}_{,1} T^{\rho\sigma}{}_{\mu}\left(T_{\nu\rho\sigma} - 2T_{[\rho\sigma]\nu}\right) - \frac{1}{2}\mathcal{F}_{,3} T^{\sigma}{}_{\sigma\rho}\left(T^{\rho}{}_{\mu\nu} + 2T_{(\mu\nu)}{}^{\rho}\right) \\ & + \frac{1}{2}\mathcal{F}_{,2}\left[T_{\mu}{}^{\rho\sigma}\left(2T_{\rho\sigma\nu} - T_{\nu\rho\sigma}\right) + T^{\rho\sigma}{}_{\mu}\left(2T_{[\rho\sigma]\nu} - T_{\nu\rho\sigma}\right)\right].\end{aligned}$$

PPN parameters

- PPN parameters [Ualikhanova, MH '19]:

$$\beta - 1 = -\frac{\varepsilon}{2}\,, \quad \gamma - 1 = -2\varepsilon\,, \quad \varepsilon = \frac{2F_{,1} + F_{,2} + F_{,3}}{2(2F_{,1} + F_{,2} + 2F_{,3})}\,,$$

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$$\eta_N = 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2 \equiv 0\,.$$

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- Bounds on theory parameters from Cassini tracking:

$$\gamma - 1 = -2\varepsilon = (2.1 \pm 2.3) \cdot 10^{-5}\,.$$

Axial-vector-tensor decomposition

- Irreducible decomposition of torsion components:

$$T_{\text{ax}} = \frac{1}{18}(\mathcal{T}_1 - 2\mathcal{T}_2)\,, \quad T_{\text{ten}} = \frac{1}{2}(\mathcal{T}_1 + \mathcal{T}_2 - \mathcal{T}_3)\,, \quad T_{\text{vec}} = \mathcal{T}_3\,.$$

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$$F_{,1} = \frac{1}{18}G_{,a} + \frac{1}{2}G_{,t}\,, \quad F_{,2} = -\frac{1}{9}G_{,a} + \frac{1}{2}G_{,t}\,, \quad F_{,3} = G_{,v} - \frac{1}{2}G_{,t}\,.$$

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⇒ Purely axial modifications do not affect PPN parameters.

Particular theories

- New general relativity [Hayashi, Shirafuji '79]:
  - Most general action linear in torsion scalars:

$$\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) = t_1\mathcal{T}_1 + t_2\mathcal{T}_2 + t_3\mathcal{T}_3 \,.$$

⇒ Taylor coefficients given by $F_{,i} = t_i$, $i = 1, \ldots, 3$.

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- $f(T)$ gravity theories [Bengochea, Ferraro '08; Linder '10]:
  - Lagrangian defined as function of linear combination:

$$\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3) = f(T)\,, \quad T = \frac{1}{4}\mathcal{T}_1 + \frac{1}{2}\mathcal{T}_2 - \mathcal{T}_3\,.$$

⇒ Taylor coefficients given by:

$$F_{,1} = \frac{1}{4}f'(0)\,, \quad F_{,2} = \frac{1}{2}f'(0)\,, \quad F_{,3} = -f'(0)$$

⇒ Indistinguishable from GR, since $\varepsilon \equiv 0$.

Outline

Action

- Gravitational part of the action:

$$S_g[\theta, \omega, \phi] = \frac{1}{2\kappa^2} \int_M \left[-\mathcal{A}(\phi) T + 2\mathcal{B}(\phi) X + 2\mathcal{C}(\phi) Y - 2\kappa^2 \mathcal{V}(\phi) \right] \theta \, \mathrm{d}^4 x \,.$$

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$$S_{\rho\mu\nu} = \frac{1}{2} \left(T_{\nu\mu\rho} + T_{\rho\mu\nu} - T_{\mu\nu\rho}\right) - g_{\rho\mu} T^{\sigma}{}_{\sigma\nu} + g_{\rho\nu} T^{\sigma}{}_{\sigma\mu} \,.$$

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- Taylor expansion of functions around background $\overset{0}{\phi} = \Phi$:

$$A = \mathcal{A}(\Phi)\,, \quad A' = \mathcal{A}'(\Phi)\,, \quad A'' = \mathcal{A}''(\Phi)\,, \quad A''' = \mathcal{A}'''(\Phi)\,, \quad \ldots$$

Field equations

- Field equations:
  - Tetrad field equations - symmetric part:

$$\kappa^2\Theta_{\mu\nu} = (\mathcal{A}' + \mathcal{C})\, S_{(\mu\nu)}{}^{\rho}\phi_{,\rho} + \mathcal{A}\left(\overset{\circ}{R}_{\mu\nu} - \frac{1}{2}\overset{\circ}{R}g_{\mu\nu}\right) - (\mathcal{B} - \mathcal{C}')\phi_{,\mu}\phi_{,\nu}$$
$$+ \left(\frac{1}{2}\mathcal{B} - \mathcal{C}'\right)\phi_{,\rho}\phi_{,\sigma}g^{\rho\sigma}g_{\mu\nu} + \mathcal{C}\left(\overset{\circ}{\nabla}_{\mu}\overset{\circ}{\nabla}_{\nu}\phi - \overset{\circ}{\square}\phi g_{\mu\nu}\right) + \kappa^2\mathcal{V}g_{\mu\nu}\,.$$

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  - Trace reversal: $\overset{\circ}{R}_{\mu\nu} - \frac{1}{2}\overset{\circ}{R}g_{\mu\nu} \to \overset{\circ}{R}_{\mu\nu}$.

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  - Trace reversal: $\overset{\circ}{R}_{\mu\nu} - \frac{1}{2}\overset{\circ}{R}g_{\mu\nu} \to \overset{\circ}{R}_{\mu\nu}$.
  - Eliminate second-order tetrad derivatives using tetrad equations.

Simplified field equations

- Trace-reversed symmetric tetrad field equations:

$$\bar{\Theta}_{\mu\nu} = (\mathcal{A}' + \mathcal{C}) \left(S_{(\mu\nu)}{}^{\rho} + g_{\mu\nu} T_{\chi}{}^{\chi\rho} \right) \phi_{,\rho} + \mathcal{A} \overset{\circ}{R}_{\mu\nu} + \frac{1}{2} \mathcal{C}' g_{\mu\nu} \phi_{,\rho} \phi_{,\sigma} g^{\rho\sigma} - (\mathcal{B} - \mathcal{C}') \phi_{,\mu} \phi_{,\nu} + \mathcal{C} \overset{\circ}{\nabla}_{\mu} \overset{\circ}{\nabla}_{\nu} \phi + \frac{1}{2} \mathcal{C} \overset{\circ}{\square} \phi g_{\mu\nu} - \kappa^2 \mathcal{V} g_{\mu\nu}$$

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- "Debraided" scalar field equation:

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- Trace-reversed symmetric tetrad field equations:

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⇒ Scalar source term vanishes for minimal coupling $\mathcal{C} = 0$.

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- Effective gravitational constant:

$$G_{\text{eff}} = \frac{\kappa^2}{8\pi A}\left(1 + \frac{C^2 e^{-m_\phi r}}{2AB + 3C^2}\right)$$

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- PPN parameters [Emtsova, MH '19]:

$$\gamma = 1 - \frac{C^2}{AB + 2C^2},$$

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⇒ Identical to GR $\gamma = \beta = 1$ for $C = 0$.

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- Re-obtain well-known PPN parameters as consistency check.

Minimally coupled theories

- Numerous minimally coupled ($\mathcal{C} = 0$) theories:
  - Teleparallel dark energy [Geng, Lee, Saridakis, Wu '11]:

$$\mathcal{A} = 1 + 2\kappa^2\xi\phi^2 \,, \quad \mathcal{B} = -\kappa^2 \,.$$

  - Interacting dark energy [Otalora '13]:

$$\mathcal{A} = 1 + 2\kappa^2\xi F(\phi) \,, \quad \mathcal{B} = -\kappa^2 \,.$$

  - Brans-Dicke type with a general coupling to torsion [Izumi, Gu, Ong '13]:

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⇒ Theories are indistinguishable from GR by PPN parameters.

Non-minimally coupled boundary term

- Action functional [Bahamonde, Wright '15]:

$$S_g = \int_M \left[-\frac{T}{2\kappa^2} + \frac{1}{2}\left(g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} - \xi\phi^2 T - \chi\phi^2 B \right) - \mathcal{V}(\phi) \right] \theta \mathrm{d}^4 x \,.$$

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⇒ Depends on background value Φ (determined from potential $\mathcal{V}$).

Outline

Action

- Gravitational part of the action:

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- Taylor expansion of function L around background $\overset{0}{\phi} = \Phi$:

$$L = \sum_{i,j,k,m=0}^{\infty} \frac{T^i}{i!} \frac{X^j}{j!} \frac{Y^k}{k!} \frac{(\phi - \Phi)^m}{m!} \frac{\partial^i}{\partial T^i} \frac{\partial^j}{\partial X^j} \frac{\partial^k}{\partial Y^k} \frac{\partial^m}{\partial \phi^m} L \bigg|_{T=X=Y=0, \phi=\Phi} \, .$$

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- Taylor coefficients $l_0, l_T, l_X, l_Y, l_\phi, \ldots$.

Field equations

- Tetrad field equations:

$$
\begin{aligned}
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&\quad\quad + L_Y\left(T_{(\mu\nu)}{}^{\rho}\phi_{,\rho} + \frac{1}{2}T^{\rho}{}_{\mu\nu}\phi_{,\rho} + T^{\rho}{}_{\rho\mu}\phi_{,\nu}\right) - Lg_{\mu\nu} = 2\kappa^2\Theta_{\mu\nu}\,.
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$$- L_T\left(T^\rho{}_{\rho\sigma}T^\sigma{}_{\mu\nu} + 2T^\rho{}_{\rho\sigma}T_{(\mu\nu)}{}^\sigma - \frac{1}{2}T_{\mu\rho\sigma}T_\nu{}^{\rho\sigma} + T_{\mu\rho\sigma}T^{\rho\sigma}{}_\nu\right)$$
$$+ L_Y\left(T_{(\mu\nu)^\rho\phi_{,\rho}} + \frac{1}{2}T^\rho{}_{\mu\nu}\phi_{,\rho} + T^\rho{}_{\rho\mu}\phi_{,\nu}\right) - Lg_{\mu\nu} = 2\kappa^2\Theta_{\mu\nu}\,.$$

- Scalar field equation:

$$g^{\mu\nu}\mathring{\nabla}_\mu\left(L_Y T^\rho{}_{\rho\nu} - L_X\phi_{,\nu}\right) - L_\phi = 0\,.$$

PPN parameters

- PPN parameters [Flathmann, MH '19]:

$$\gamma - 1 = \frac{l_Y{}^2}{2l_T l_X - 2l_Y^2}\,,$$

$$\beta - 1 = \frac{l_Y\left[l_T l_X l_Y^2\left(16l_{T\phi} - 7l_Y\right) + 3l_Y^4\left(l_Y - 2l_{T\phi}\right) - 8l_T^2 l_X^2 l_{T\phi} + 2l_T^2 l_Y\left(2l_X^2 + l_Y l_{X\phi} - 2l_X l_{Y\phi}\right)\right]}{8\left(4l_T l_X - 3l_Y^2\right)\left(l_Y^2 - l_T l_X\right)^2}\,,$$

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⇒ Identical to GR $\gamma = \beta = 1$ for $l_Y = 0$.

Analogue of scalar-curvature gravity

- Consider previous studied class of theories:

$$L(T, X, Y, \phi) = -\mathcal{A}(\phi)T + 2\mathcal{B}(\phi)X + 2\mathcal{C}(\phi)Y - 2\kappa^2\mathcal{V}(\phi)\,.$$

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⇒ Consistency of both derivations.

Theories without derivative couplings

- Consider simple scalar-torsion theory [MH, Järv, Ualikhanova '18]:

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⇒ Theories are indistinguishable from GR $\gamma = \beta = 1$.

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- Post-Newtonian limit of teleparallel gravity theories:
  - Obtained PPN parameters for different teleparallel theories.
  - All considered theories are fully conservative.
  - Large, widely used subclasses have same PPN limit as GR:
    - $f(T)$ theories with torsion scalar T from TEGR.
    - Minimally coupled scalar-torsion theories with $C = 0$ or $l_Y = 0$.
  - $\mathcal{F}(\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$ theories show no Nordvedt effect.

Outlook and work in progress

- Consider more general teleparallel gravity theories:
  - Theories with modified constitutive laws [MH, Järv, Krššák, Pfeifer '17].
  - Teleparallel extension to Horndeski gravity [Bahamonde, Dialektopoulos, Said '19].
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- Extend formalism by including higher perturbation orders:
  - General covariant expansion instead of space-time split.
  - Allow also for fast-moving source masses.
  - Consider inspiral phase of black hole merger event.
  - Devise method for calculating gravitational waves.

Literature

- U. Ualikhanova and MH,
  "Parameterized post-Newtonian limit of general teleparallel gravity theories",
  arXiv:1907.08178 [gr-qc].
- E. D. Emtsova and MH,
  "Post-Newtonian limit of scalar-torsion theories of gravity as analogue to scalar-curvature theories",
  arXiv:1909.09355 [gr-qc].
- K. Flathmann and MH,
  "Post-Newtonian limit of generalized scalar-torsion theories of gravity",
  arXiv:1910.01023 [gr-qc].

Prospects for students

- Possibilities for study / research visits to Tartu.
  - Courses on gravity and differential geometry in English.
  - Active research group in teleparallel and related theories.
  - Large share of students - positive social environment.
- Exchange programs for master and PhD students.
- More information: `http://www.studyinestonia.ee`.