Übungen zur Supersymmetrie - WS06/07

Blatt 1 - 2. November 2006
Besprechung am 9. November 2006

1. Zeigen Sie:
   - $$\psi\chi = \chi\psi$$
   - $$\chi\sigma^n\bar{\psi} = -\bar{\psi}\bar{\sigma}^n\chi$$
   - $$(\chi\sigma^m\bar{\psi})^\dagger = \psi\sigma^m\bar{\chi}$$
   - $$(\chi\sigma^m\bar{\sigma}^n\psi)^\dagger = \bar{\psi}\bar{\sigma}^n\sigma^m\bar{\chi}$$
2. Eliminieren Sie das Hilfsfeld F aus der Lagrangedichte
   $$\mathcal{L} = \mathcal{L}_0 + m\mathcal{L}_m$$
   $$\mathcal{L}_0 = i\partial_n\bar{\psi}\bar{\sigma}^n\psi + A^*\Box A + F^*F$$
   $$\mathcal{L}_m = AF + A^*F^* - \frac{1}{2}\psi\psi - \frac{1}{2}\bar{\psi}\bar{\psi}$$
   um die Form
   $$\mathcal{L} = i\partial_n\bar{\psi}\bar{\sigma}^n\psi - \frac{1}{2}m(\psi\psi + \bar{\psi}\bar{\psi}) + A^*\Box A - m^2A^*A$$
   zu erhalten.
3. Berechnen Sie $\left\{D_\alpha, \bar{D}_{\dot{\alpha}}\right\}$ unter Verwendung von
   $$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma_{\alpha\dot{\alpha}}{}^m\bar{\theta}^{\dot{\alpha}}\partial_m$$
   $$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^\alpha\sigma_{\alpha\dot{\alpha}}{}^m\partial_m$$
4. Zeigen Sie, dass die Weyl-Basis
   $$\gamma_W^m = \begin{pmatrix} 0 & \sigma^m \\ \bar{\sigma}^m & 0 \end{pmatrix}$$
   und die Majorana-Basis
   $$\gamma_M^0 = \begin{pmatrix} 0 & -\sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}, \quad \gamma_M^1 = \begin{pmatrix} 0 & i\sigma^3 \\ i\sigma^3 & 0 \end{pmatrix},$$
   $$\gamma_M^2 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma_M^3 = \begin{pmatrix} 0 & -i\sigma^1 \\ -i\sigma^1 & 0 \end{pmatrix}$$
   zusammenhängen über die Transformation
   $$\gamma_M^m = Y\gamma_W^mY^{-1}, \quad Y = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \epsilon_{\dot{\alpha}\dot{\beta}} \\ -i & i\epsilon_{\dot{\alpha}\dot{\beta}} \end{pmatrix}$$
   Dabei ist
   $$\sigma^0 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
   $$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
   und $\bar{\sigma}^{m\dot{\alpha}\alpha} = \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\sigma^m_{\beta\dot{\beta}}$.