Selected Topics in the Theories of Gravity - Assignment 5

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1. **Lie bracket of Killing vector fields**
   Consider two Killing vector fields $\xi = \xi^\mu \partial_\mu$ and $\xi' = \xi'^\mu \partial_\mu$.

   (a) Calculate the components ζ^μ of the the Lie bracket $\zeta = [\xi, \xi']$. Recall that the Lie bracket can be defined by its operation on scalar fields ϕ,

   $$[\xi, \xi']\phi = \xi(\xi'\phi) - \xi'(\xi\phi)\,. \tag{1}$$

   (b) Show that ζ is also a Killing vector field.

2. **Cylindrical symmetry**
   Consider the metric

   $$ds^2 = -A(\rho)dt^2 + B(\rho)dz^2 + C(\rho)(x\,dx + y\,dy)^2 + D(\rho)(y\,dx - x\,dy)^2 \tag{2}$$

   with $\rho^2 = x^2 + y^2$ and the diffeomorphisms f_{ϕ,c_t,c_z} given by

   $$t' = t + c_t\,, \quad z' = z + c_z\,, \quad x' = x\cos\phi + y\sin\phi\,, \quad y' = -x\sin\phi + y\cos\phi\,. \tag{3}$$

   (a) Show that f_{ϕ,c_t,c_z} is an isometry.

   (b) Find the orbits of this isometry group. What do they look like?

   (c) Determine the Killing vector fields of the 1-parameter subgroups $f_{\phi,0,0}$, $f_{0,c_t,0}$ and $f_{0,0,c_z}$.