Selected Topics in the Theories of Gravity - Assignment 6

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1. **Continuity equation**

   (a) Show that the continuity equation

   $$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p) \tag{1}$$

   follows from the covariant energy-momentum conservation $\nabla_\mu T^{\mu 0} = 0$ of

   $$T_{00} = \rho\,, \quad T_{0i} = 0\,, \quad T_{ij} = pa^2\gamma_{ij}\,. \tag{2}$$

   (b) Use the first Friedmann equation

   $$3\frac{k + \dot{a}^2}{a^2} - \Lambda = 8\pi G\rho \tag{3}$$

   to show that (1) is equivalent to the second Friedmann equation

   $$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}\,. \tag{4}$$

   (c) Show that

   $$\rho = \frac{3H_0^2}{8\pi G}\sum_w \Omega_w \left(\frac{a_0}{a}\right)^{3w+3} \tag{5}$$

   satisfies (1), where the sum runs over different barotropic indices w and Ω_w are constants.

2. $\mathbf{w = -1/3}$
   Discuss a universe with perfect fluid matter with barotropic index $w = -1/3$. Solve the Friedmann equations for $k = -1, 0, 1$. What does $a(t)$ look like?